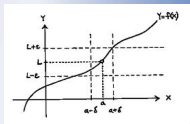
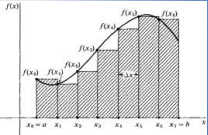


23 Differential Equations



$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$



$$\lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n f(x_i) \Delta x_i = \int_a^b f(x) dx$$

$$\int_a^b f(x) dx = F(b) - F(a)$$

Differential Equations

EXAMPLE: $\frac{dy}{dx} = \frac{x^2}{y}$

SOLUTION: $y dy = x^2 dx$
 $\int y dy = \int x^2 dx$
 $\frac{1}{2} y^2 = \frac{1}{3} x^3 + C$
 $y^2 = \frac{2}{3} x^3 + C$
 $y = \pm \sqrt{\frac{2}{3} x^3 + C}$

A **differential equation** is an equation that contains a derivative. We will need to integrate both sides, at some point, to 'undo' the derivative.

- EX 1 Find the equation of the curve that goes through the point (2,-4) and whose slope at any point on the curve is $3x$.

23 Differential Equations

EX 2 $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y = 4$ when $x = 1$

EX 3 $\frac{dy}{dx} = -y^2(x^2 + 2)^4$ through $(0, 1)$

23 Differential Equations

- EX 4 The acceleration of an object moving along a coordinate line is $a(t)=18(t-3)^{-3}$ in meters per second per second.
- a) If the velocity at $t=0$ is 4 meters per second, find the velocity 2 seconds later.
- b) If the initial position is -3 m, find an equation for the position of the object at time, t .

EXAMPLE: $\frac{dy}{dx} = \frac{x^2}{y}$

SOLUTION: $ydy = x^2dx$
 $\int ydy = \int x^2dx$
 $\frac{1}{2}y^2 = \frac{1}{3}x^3 + C$
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