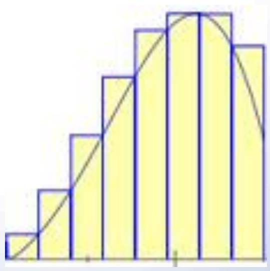
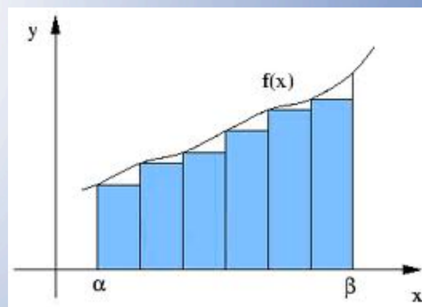
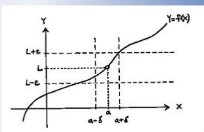
Introduction to Area

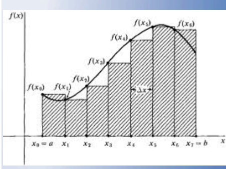






$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

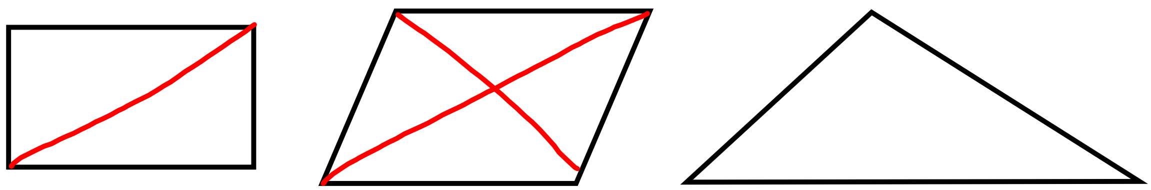
$$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$$

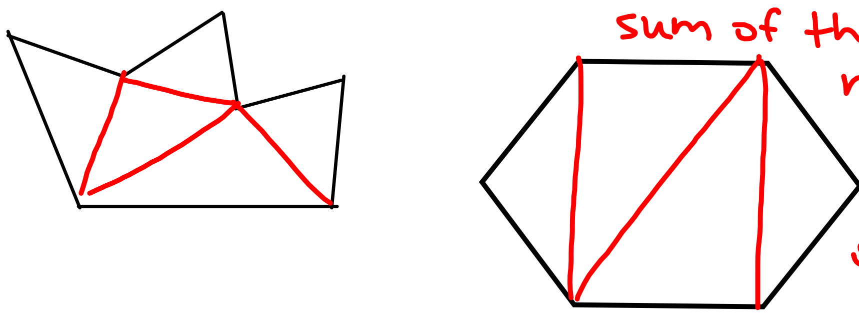


$$\lim_{max\,\Delta x_i \to 0} \sum_1^n f(x_i)\Delta x_i = \int_a^b f(x)dx$$

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

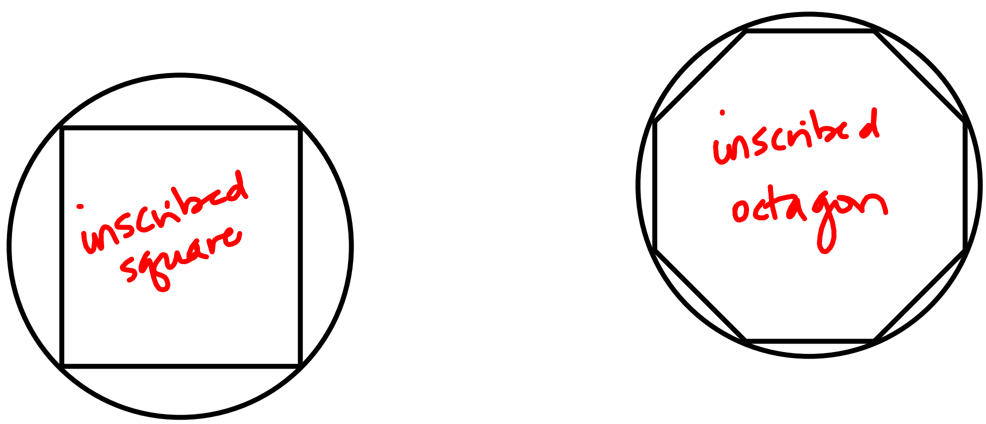
Area of a Polygon:

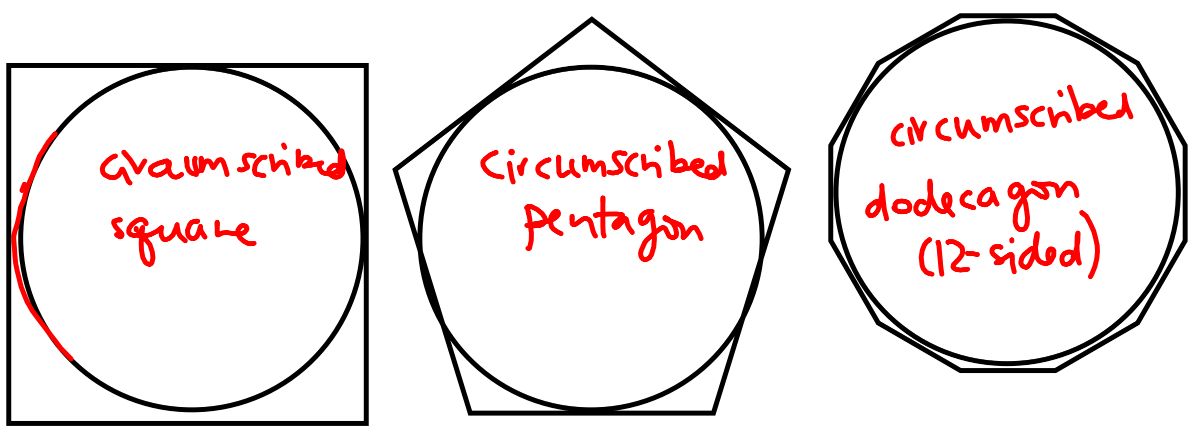




area of entire shape = sum of the areas of nonoverlapping shapes inside original polygon

Estimating the area of a circle:





Sums and Sigma Notation

$1+2+3+4+...+ 100 = \sum_{i=1}^{100} i$

$2+4+6+8+....+ 1000 = \sum_{j=1}^{500} (2j)$

j=1 j=2 j=3 j=4

$1+4+9+16+....+ 625= \sum_{j=1}^{25} j^2$

j=1 j=2 j=3 j=4

$\boxed{\sum =}$ Sigma, the capitol Greek letter called "sigma";

It means summation. i = index

1. $\sum_{j=1}^{n} \frac{1}{j} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{n}$

2. $\sum_{i=1}^{n} c = \underbrace{c + c + c + \dots + c}_{n \text{ times}} = nc$

Linearity of $\sum$ (finite sum)

Let $\{a_i\}$ *and* $\{b_i\}$ denote two sequences and *c* is a real number.

(i) $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$ (we can factor out a constant from the sum)

(ii) $\sum_{i=1}^{n} a_i \pm b_i = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

(distributes thru addition/subtraction)

Special Sum Formulas

$$\sum_{i=1}^{n} i = 1+2+3+\ldots+n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

these should go on your note card

$$\sum_{i=1}^{n} i^3 = 1^3+2^3+3^3+\ldots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{i=1}^{n} i^4 = 1^4+2^4+3^4+\ldots+n^4 = \frac{n(n+1)(6n^3+9n^2+n-1)}{30}$$

ex $1+2+3+4+5+\ldots+98+99+100 = 101(50)$

$= 5{,}050$

$1+100 = 101$

$2+99 = 101$

$3+98 = 101$

$\vdots$

$50+51 = 101$

$1+2+3+\ldots+n = (n+1)\frac{n}{2}$

$1+n = 1+n$

$2+(n-1) = 1+n$

$3+(n-2) = 1+n$

$\vdots$

EX 1 $\sum_{i=1}^{10}[(i-1)(4i+3)]$ $= \sum_{i=1}^{10}(4i^2+3i-4i-3)$

$= \sum_{i=1}^{10}(4i^2-i-3) = 4\sum_{i=1}^{10} i^2 - \sum_{i=1}^{10} i - 3\sum_{i=1}^{10} 1$

$= \cancel{4}^{2}\left(\frac{10(10+1)(2(10)+1)}{\cancel{6}_{3}}\right) - \left(\frac{10(10+1)}{2}\right) - 3(10)$

$= \frac{20(11)\cancel{(21)}^{7}}{\cancel{3}} - 5(11) - 30 = 135(11) - 30$

$= 1455$

EX 2 $\sum_{j=1}^{n}(2j-3)^2$

$= \sum_{j=1}^{n}(4j^2-12j+9) = 4\sum_{j=1}^{n} j^2 - 12\sum_{j=1}^{n} j + \sum_{j=1}^{n} 9$

$= 4\left(\frac{n(n+1)(2n+1)}{6}\right) - 12\left(\frac{n(n+1)}{2}\right) + 9n$

EX 5 Change the variable in the index to start at 1. $\sum_{k=5}^{14} k2^{k-4}$

$j = k-4$ k starts at 5, want j to start at 1

$\Rightarrow k=5, j=1$

$k=14, j=14-4=10$

$k = j+4$

$\sum_{k=5}^{14} k2^{k-4} = \boxed{\sum_{j=1}^{10}(j+4)2^{j}}$

Collapsing Sum $\sum_{i=1}^{n}(a_{i+1}-a_i)=a_{n+1}-a_1$

*note: it will be the sum of a difference where the two terms look the same except w/ different counters

EX 3 $\sum_{k=1}^{10}(2^k-2^{k-1})$

$$=\underbrace{(\cancel{2^1}-2^0)}_{k=1}+\underbrace{(\cancel{2^2}-\cancel{2^1})}_{k=2}+\underbrace{(\cancel{2^3}-\cancel{2^2})}_{k=3}+\dots+\underbrace{(\cancel{2^9}-\cancel{2^8})}_{k=9}+\underbrace{(2^{10}-\cancel{2^9})}_{k=10}$$

$$=-2^0+2^{10}=-1+2^{10}=1023$$

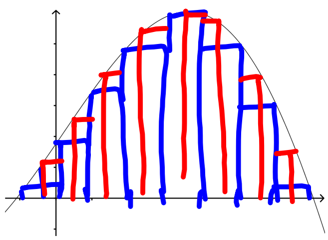
EX 4 $\sum_{k=3}^{m+1}(a_k-a_{k-1})$

$$=(\cancel{a_3}-a_2)+(\cancel{a_4}-\cancel{a_3})+(\cancel{a_5}-\cancel{a_4})+\dots+\underbrace{(\cancel{a_m}-\cancel{a_{m-1}})}_{k=m}+\underbrace{(a_{m+1}-\cancel{a_m})}_{k=m+1}$$

$$=\boxed{-a_2+a_{m+1}}$$

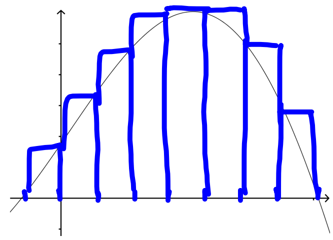
We will estimate the area under a curve using inscribed or circumscribed rectangles.

inscribed rectangles



under estimate area under curve

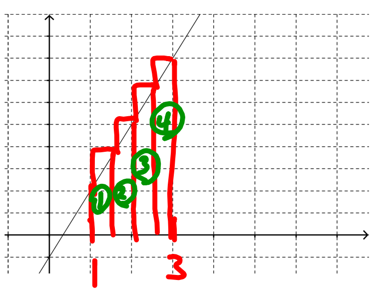
circumscribed rectangles.



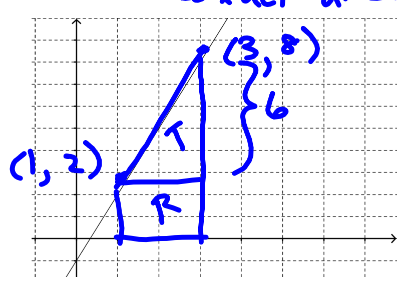
over estimate area under curve

EX 6 For f(x)=3x-1, divide the interval [1,3] into 4 equal subintervals.

Calculate the area of the circumscribed rectangles.



exact area



$A_R = 2(2) = 4$

$A_T = \frac{1}{2}(2)(6)$
$= 6$

$A_{entire} = 10$

$x_1 = 1,\ x_2 = 1.5,\ x_3 = 2,\ x_4 = 2.5,\ x_5 = 3$

$f(x) = 3x - 1$

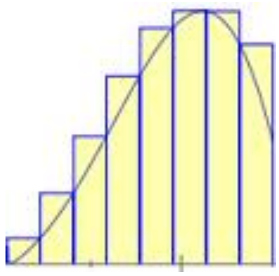
$A_1 = \frac{1}{2}(f(1.5)) = \frac{1}{2}(3(\frac{3}{2}) - 1) = \frac{1}{2}(\frac{7}{2}) = 7/4$

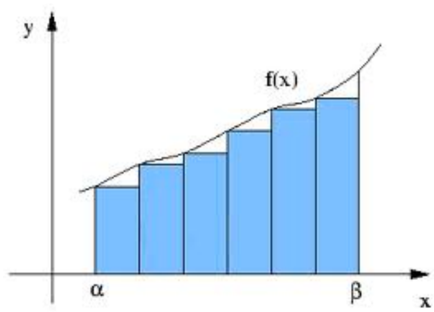
$A_2 = \frac{1}{2}(f(2)) = \frac{1}{2}(3(2) - 1) = \frac{1}{2}(5) = 5/2$

$A_3 = \frac{1}{2}(f(2.5)) = \frac{1}{2}(3(\frac{5}{2}) - 1) = \frac{1}{2}(\frac{13}{2}) = \frac{13}{4}$

$A_4 = \frac{1}{2}(f(3)) = \frac{1}{2}(3(3) - 1) = 4$

total area $= \frac{7}{4} + \frac{5}{2} + \frac{13}{4} + 4$
$= \frac{20}{4} + \frac{5}{2} + 4$
$= 9 + \frac{5}{2} = 11\frac{1}{2}$




y
f(x)
α
β
x