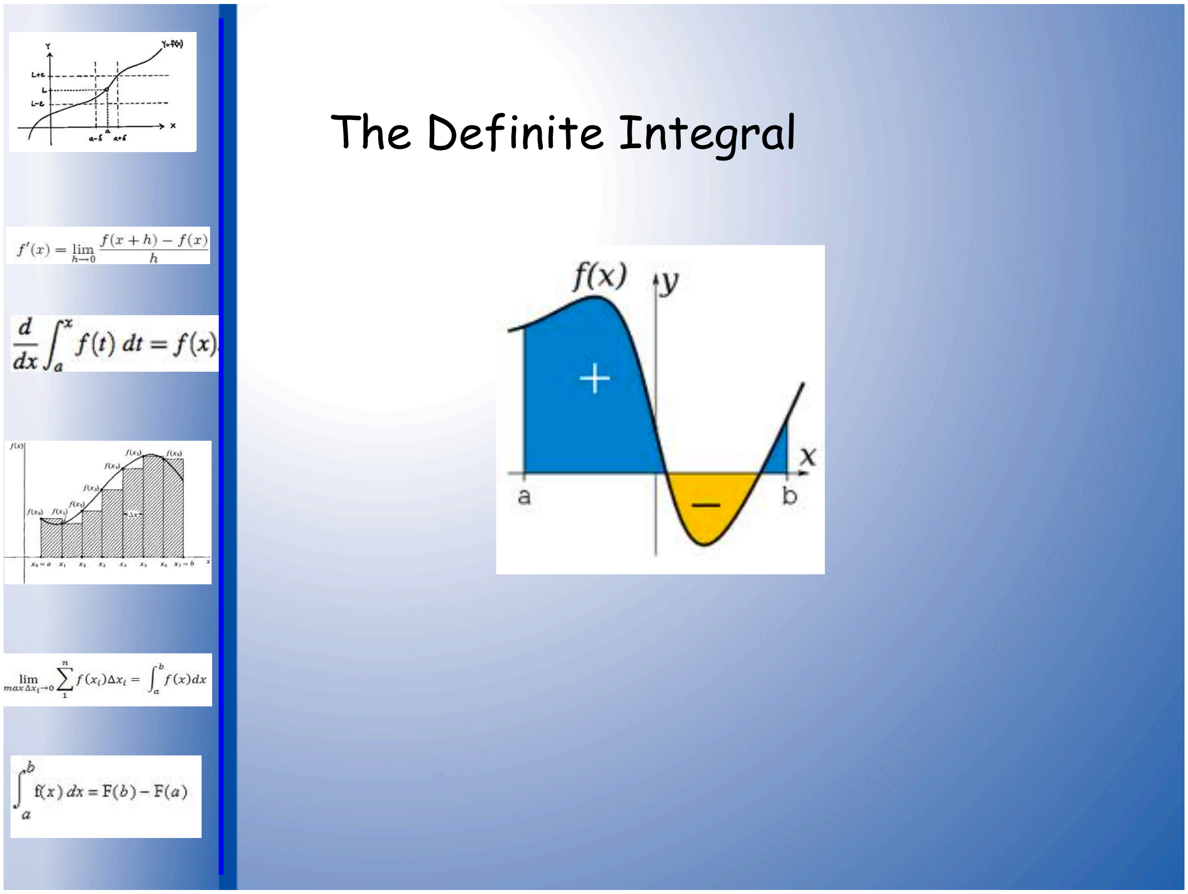
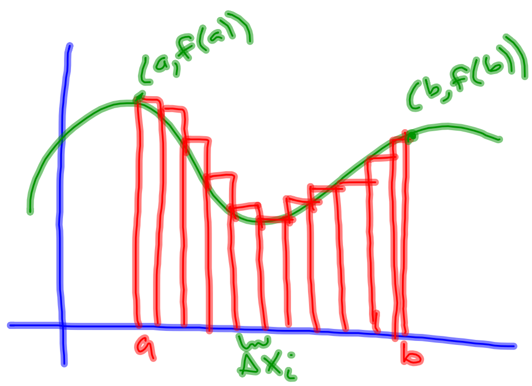


The Definite Integral



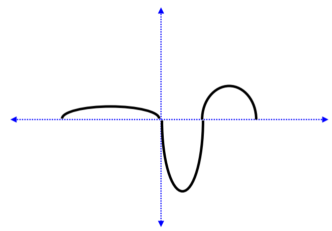
25 Definite Integral

Definition of the Definite Integral

Let *f* be a function that is defined on [a,b]. If $\lim_{|P|\to 0}\sum_{i=1}^{n} f(\overline{x}_i)\Delta x_i$ exists,

we say *f* is integrable on [a,b] and $\int_a^b f(x)dx = \lim_{|P|\to 0}\sum_{i=1}^{n} f(\overline{x}_i)\Delta x_i$.

$$\int_a^b f(x)dx = A_{up} - A_{down}$$



$$\int_a^a f(x)dx = 0$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

Integrability Theorem

If *f* is bounded on [a,b] and continuous there except for a finite number of discontinuities, then *f* is integrable on [a,b]. So, if *f* is continuous on [a,b] it is integrable on [a,b].

Interval Additive Property

If *f(x)* is integrable, then $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$.

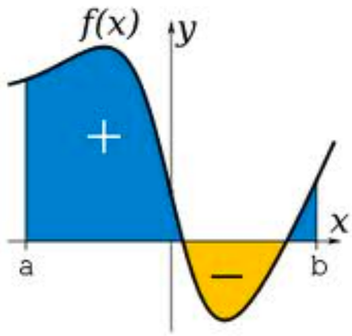
EX 1 Evaluate this definite integral using the definition.

$$\int_{-1}^{2} (2x-3)\, dx$$

EX 2 Evaluate this definite integral using the definition.

$$\int_0^2 (3x^2 + 2)dx$$

EX 3 Find the area of the region under the curve of $f(x) = -x^2 + 1$ on the interval [-1,1].
(To do this, divide the interval [-1,1] into n equal subintervals,
calculate the area of the circumscribed or inscribed rectangles
and take the limit as $n \to \infty$.)


f(x)
y
+
x
a
−
b