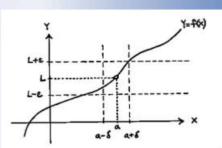
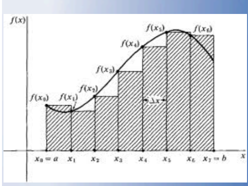
# The First Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$



$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$



$$\lim_{\max \Delta x_i \to 0}\sum_{1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$$

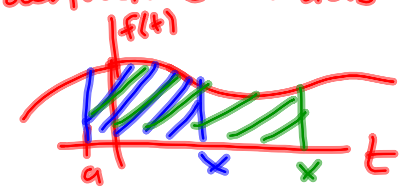
$$\int_a^b f(x)\,dx = F(b) - F(a)$$

**The First Fundamental Theorem of Calculus**

Let $f$ be continuous on [a,b] and let $x$ be a value in (a,b). Then

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

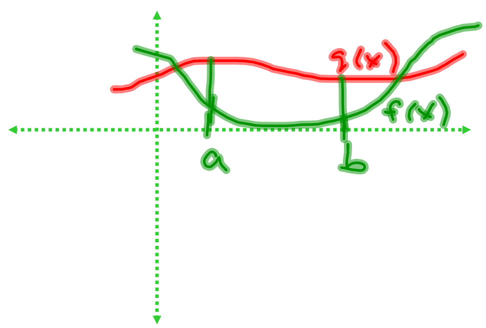
(derivative "undoes" accumulation fn)




**Theorem Comparison Property**

If $f$ and $g$ are integrable on [a,b] and if $f(x) \leq g(x)$ for all $x$ on [a,b],

then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$

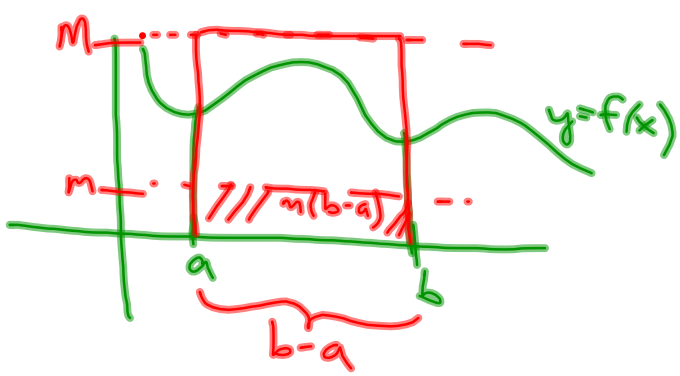



**Theorem Boundless Property**

If f is integrable on [a,b] and $m \leq f(x) \leq M$ for all a on [a,b],

then $m(b\text{-}a) \leq \int_a^b f(x)dx \leq M(b\text{-}a)$

m (b-a) = area of
rectangle w/ length
m & width b-a



**Theorem** **Linearity of the Definite Integral**

If *f* and *g* are integrable on [a,b] and *k* is a real number, then

(i) $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ — we can factor a constant factor

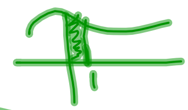
and

(ii) $\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$ — distribute def. integral through addition/subtraction

EX 1 Suppose $\int_0^1 f(x)dx = 2$ $\quad \int_1^2 f(x)dx = 3$

$\int_0^1 g(x)dx = -1$ $\quad \int_0^2 g(x)dx = 4$

$\int_0^1 f(x)dx = 2$ means the area under curve $y=f(x)$ (& above x-axis) is 2, from $x=0$ to $x=1$



Calculate $\int_0^2 (\sqrt{3}f(t) + \sqrt{2}g(t) + \pi)dt$ .

$$= \sqrt{3}\int_0^2 f(t)\,dt + \sqrt{2}\int_0^2 g(t)\,dt + \pi\int_0^2 dt$$

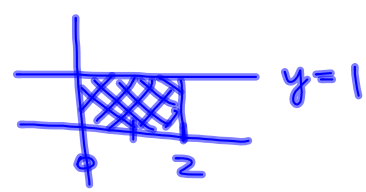
$$= \sqrt{3}\left(\int_0^1 f(t)dt + \int_1^2 f(t)dt\right) + \sqrt{2}(4) + \pi\int_0^2 dt$$

$$= \sqrt{3}(2+3) + 4\sqrt{2} + 2\pi$$

$$= \boxed{5\sqrt{3} + 4\sqrt{2} + 2\pi} \text{ units}^2$$

aside:

$\pi\int_0^2 dt = \pi\int_0^2 1\,dt$




area: $= \int_0^2 1\,dt = 2(1) = 2$

$\Rightarrow \pi\int_0^2 1\,dt = 2\pi$

EX 2 Find *G'(x)* for each of these.

note: $\int_a^b f(t)\,dt = \int_a^b f(x)\,dx = \int_a^b f(p)\,dp$

a) *G(x)* = $\int_3^x 4t\,dt$

$G'(x) = \frac{d}{dx}\left(\int_3^x 4t\ dt\right)$

$= 4x$

b) *G(x)* = $\int_1^x (\cos^3(2t)\tan(t))\,dt \qquad -\pi/2 < x < \pi/2$

$\frac{d}{dx}(G(x)) = \frac{d}{dx}\int_1^x (\cos^3(2t)\tan(t))\,dt$

$= \cos^3(2x)\tan(x)$

c) *G(x)* = $\int_{-2}^x (xt)\,dt$

$G(x) = x\int_{-2}^x t\,dt$

$G'(x) = 1\cdot\int_{-2}^x t\,dt + x\cdot\frac{d}{dx}\left(\int_{-2}^x t\,dt\right)$

$= \int_{-2}^x t\,dt + x(x)$

$= \int_{-2}^x t\,dt + x^2$

EX 3 Find $\frac{d}{dx}\int_1^{x^2+x}\sqrt{2w+\sin w}\,dw$

(need chain rule)

$$\frac{d}{dx}\left(\int_1^{x^2+x}\sqrt{2w+\sin w}\,dw\right)$$

$$=\frac{d}{d(x^2+x)}\left(\int_1^{x^2+x}\sqrt{2w+\sin w}\,dw\right)\left(\frac{d(x^2+x)}{dx}\right)$$

$$=\sqrt{2(x^2+x)+\sin(x^2+x)}\,(2x+1)$$

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$