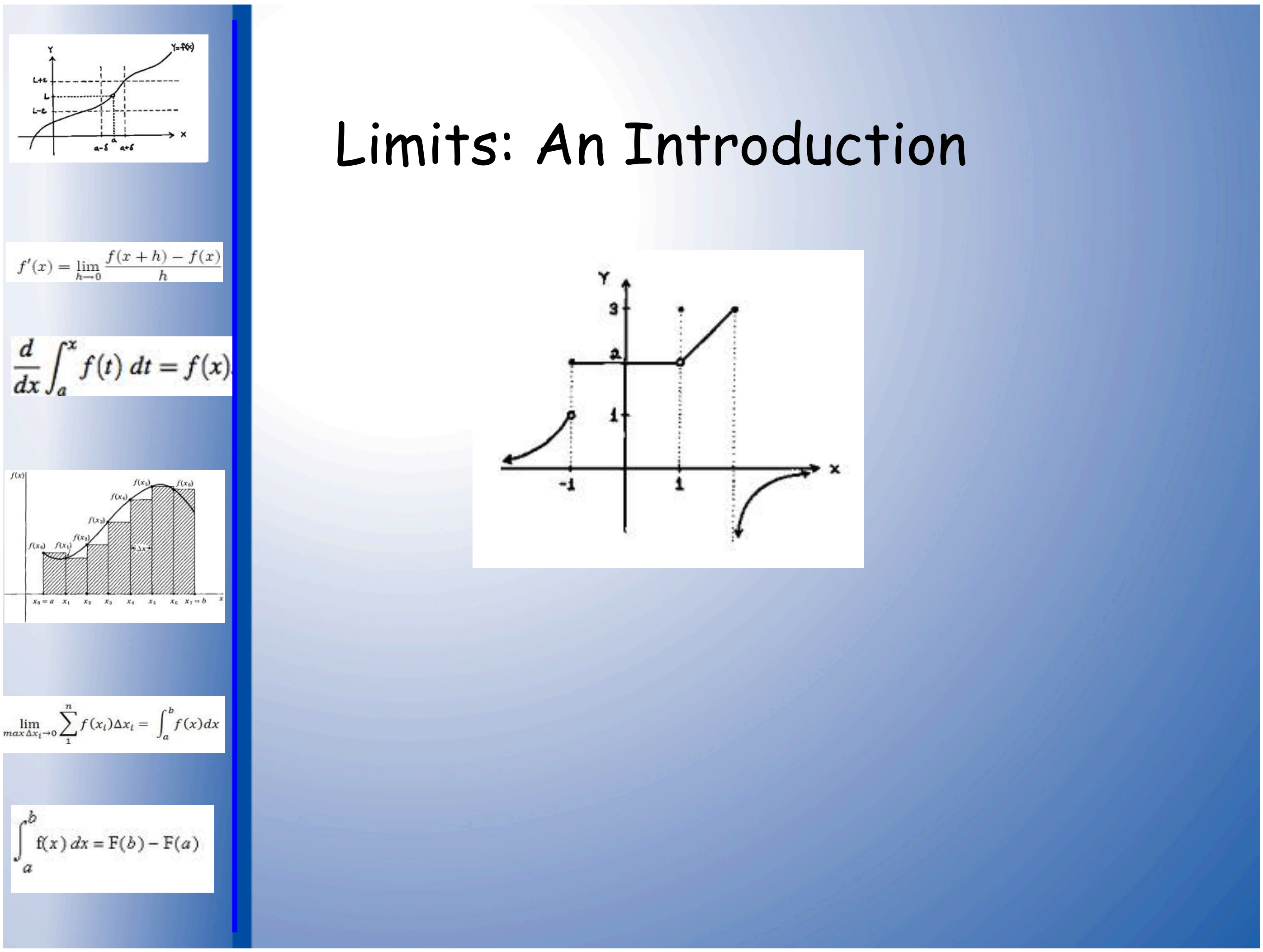

Limits: An Introduction
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$
$\lim_{max\, \Delta x_i \to 0} \sum_1^n f(x_i) \Delta x_i = \int_a^b f(x) dx$
$\int_a^b f(x)\, dx = F(b) - F(a)$
Y
X
3
2
1
-1
1

2B Introduction to Limits

Consider this function: $f(x)=\dfrac{x^2+x-12}{x-3}$ if $x=3$, we get " $\frac{0}{0}$ case"

What happens at x = 3? at $x=3$, the denominator is 0.

What happens as we approach x = 3?

x	f(x)
3.25	7.25
3.2	7.2
3.1	7.05
3.05	7.01
3.001	7.001
3	?
2.00	6.99
2.95	6.95
2.9	6.9
2.8	6.8

So we say as x approaches 3, f(x) approaches 7.

y-value goes to 7 as $x \to 3$ "x goes to 3"

Algebraically we compute it this way:

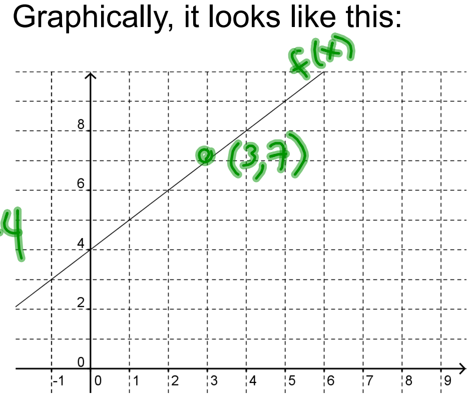
$$\frac{x^2+x-12}{x-3}=\frac{(x-3)(x+4)}{(x-3)}$$

$$=x+4 \Longrightarrow y=x+4$$

$$\lim_{x\to 3}\frac{x^2+x-12}{x-3}=\lim_{x\to 3}(x+4)$$

$$=3+4=7$$

Graphically, it looks like this:



2B Introduction to Limits

Definition: To say $\lim_{x\to c} f(x) = L$ means that when *x* is near, but different from *c*, then *f(x)* is near *L*.

"the limit of f(x) as x approaches c is L"

Ex 1 $\lim_{x\to 2}(3x+1) = 3(2)+1 = 7$

the graph of $y=3x+1$ goes thru $(2,7)$

Ex 2 $\lim_{x\to 5}\frac{2x^2-7x-15}{x-5} = \lim_{x\to 5}\frac{(x-5)(2x+3)}{(x-5)}$

plug in $x=5$: $\frac{0}{0}$ case (most interesting case) $\Rightarrow$ algebraically manipulate

$= \lim_{x\to 5} 2x+3$

$= 2(5)+3 = 13$

$y=\frac{2x^2-7x-15}{x-5}$ gets close to pt $(5,13)$ (hole there)

Ex 3 $\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3} =$

as $x=9$, $\frac{0}{0}$ case

$= \lim_{x\to 9}\left(\frac{x-9}{\sqrt{x}-3}\right)\left(\frac{\sqrt{x}+3}{\sqrt{x}+3}\right)$

$= \lim_{x\to 9}\frac{(x-9)(\sqrt{x}+3)}{(x-9)}$

$= \lim_{x\to 9}(\sqrt{x}+3) = \sqrt{9}+3 = 6$

option 2

$\lim_{x\to 9}\frac{(\sqrt{x}+3)(\sqrt{x}-3)}{\sqrt{x}-3}$

$= \lim_{x\to 9}(\sqrt{x}+3)$

this function approaches the pt $(9,6)$ (there's a hole there)

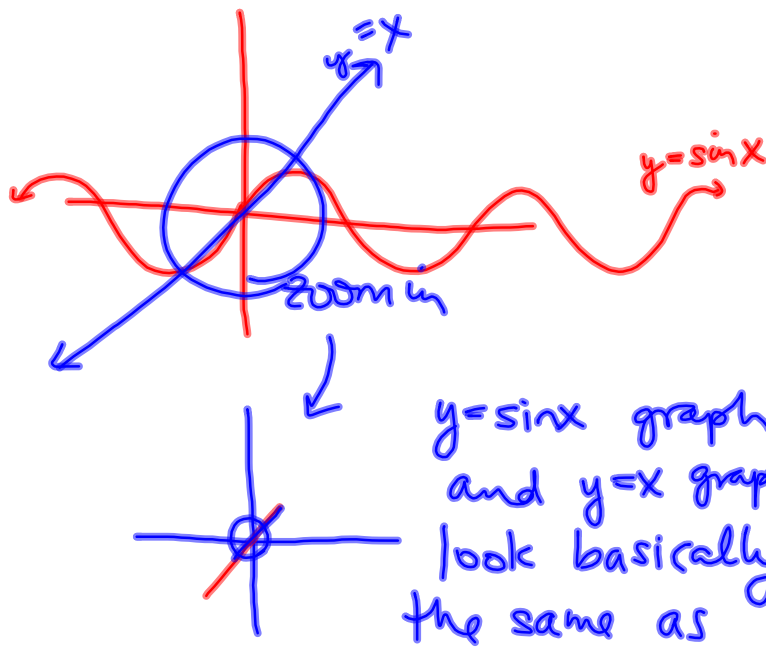
Ex 4 $\lim_{x\to 0}\frac{\sin x}{x} = 1 = \lim_{x\to 0}\frac{x}{\sin x}$

try to plug in $x=0$: $\frac{0}{0}$ case

<u>Argument 1</u>

x	$\frac{\sin x}{x}$
1.0	0.84147
0.5	0.95885
0.1	0.99833
0.01	0.99998
0	? 1
-0.01	0.99998
-0.1	0.00933
-0.5	0.05885
-1.0	0.84147

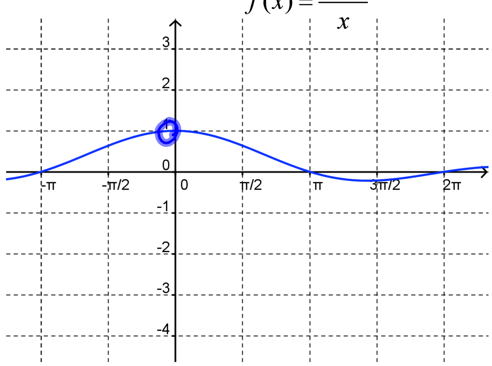
<u>Argument 2</u>



y=sinx graph and y=x graph look basically the same as x is really close to zero

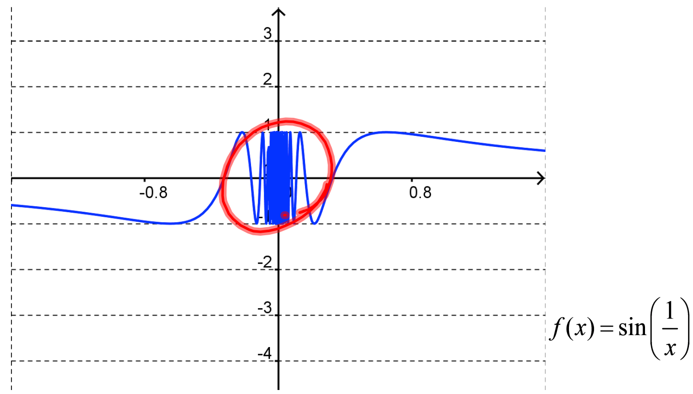
Graphically:

$f(x) = \frac{\sin x}{x}$



hole at (0,1)

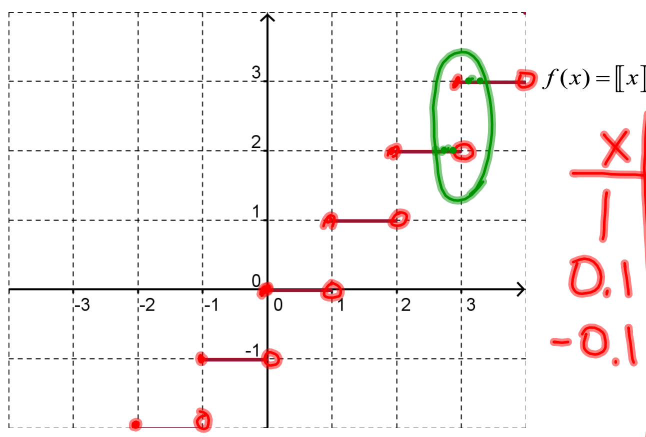
Ex 5 $\lim\limits_{x \to 0} \sin\left(\frac{1}{x}\right) =$



plug in x=0, sin of something undefined

$\lim\limits_{x \to 0} \sin\left(\frac{1}{x}\right)$ DNE

DNE "does not exist"



x	y
1	1
0.1	0
-0.1	-1

Ex 6 $\lim\limits_{x \to 3} [\![x]\!] =$

greatest integer fn

y = [[x]]

returns the biggest integer less than or equal to x.

$\lim\limits_{x \to 3} [[x]]$

DNE

problem: as x→3 from the right, the y-value is different than if x→3 from left

Definition: Right and Left Hand Limits

① $\lim_{x \to c^+} f(x) = L$ means that when x approaches *c* from the right side of *c*, then *f(x)* is near *L*.

② $\lim_{x \to c^-} f(x) = L$ means that when x approaches *c* from the left side of *c*, then *f(x)* is near *L*.

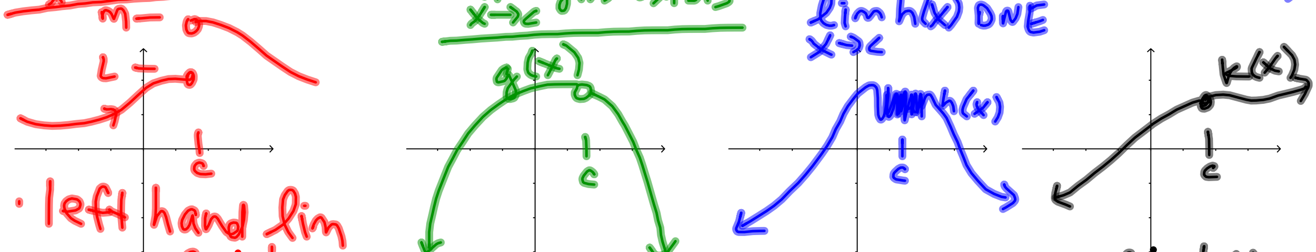
iff = if and only if (the implication goes both ways)

Theorem A $\lim_{x \to c} f(x) = L$ iff $\lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)$

$\lim_{x \to c} f(x)$ DNE

$\lim_{x \to c} g(x)$ exists

$\lim_{x \to c} h(x)$ DNE



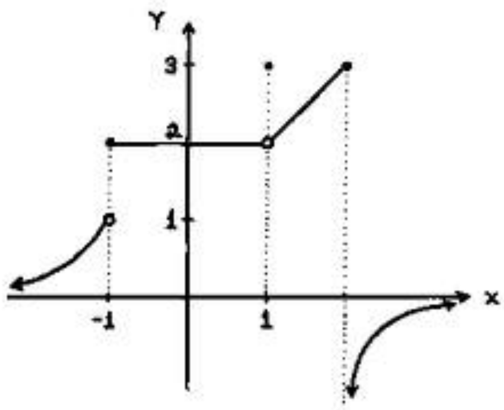
· left hand lim exists
· right hand lim exists
but $\lim_{x \to c^+} f(x) \neq \lim_{x \to c^-} f(x)$
$\lim_{x \to c^-} f(x) = L$, $\lim_{x \to c^+} f(x) = m$

$\lim_{x \to c} g(x)$ exist
g(c) DNE

there is no y-value that the graph stays at or is close to as $x \to c$

$\lim_{x \to c} k(x)$ exists and k(c) also exists

Determine these limits for this function.



$\lim_{x \to -1} f(x) =$ DNE

$\lim_{x \to -1^-} f(x) =$ 1

$\lim_{x \to -1^+} f(x) =$ 2

$\lim_{x \to 1} f(x) =$ 2

$\lim_{x \to 0} f(x) =$ 2

$\lim_{x \to 2^-} f(x) = 3$

2B Introduction to Limits