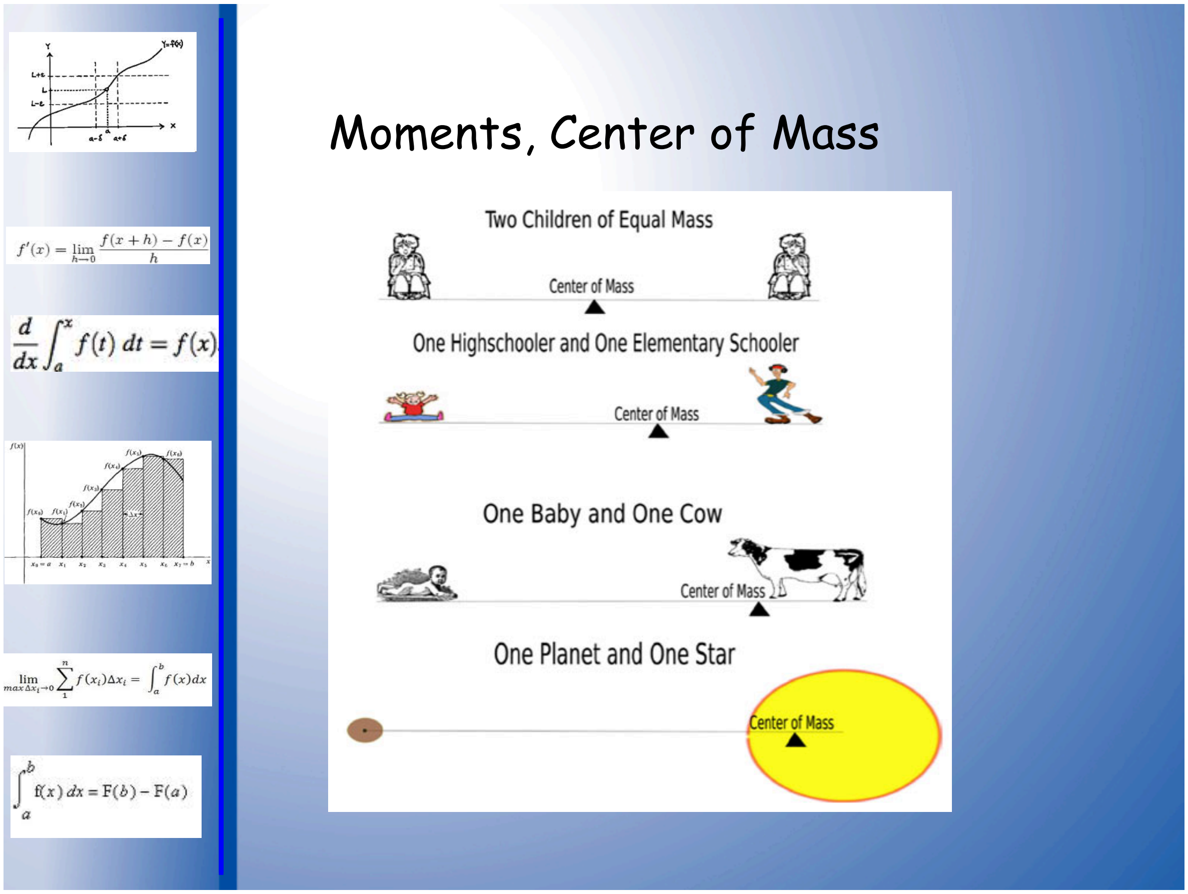
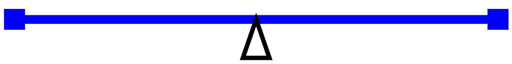
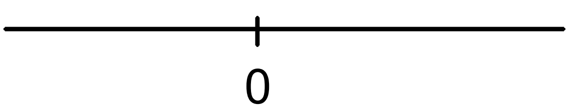


Moments, Center of Mass
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$
$\lim_{\max \Delta x_i \to 0} \sum_1^n f(x_i) \Delta x_i = \int_a^b f(x) dx$
$\int_a^b f(x)\, dx = F(b) - F(a)$
Two Children of Equal Mass
Center of Mass
One Highschooler and One Elementary Schooler
Center of Mass
One Baby and One Cow
Center of Mass
One Planet and One Star
Center of Mass





0

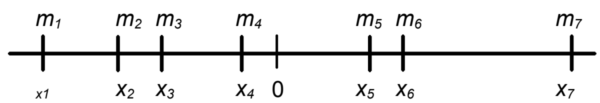
33 Moment Center of Mass

The moment of a particle with respect to a point is the product of mass (m) of the particle with its directed distance (x) from a point. This measures the tendency to produce a rotation about that point.

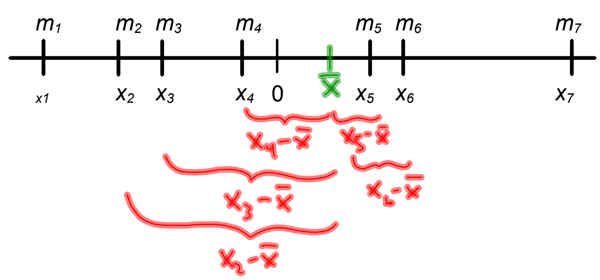
Total moment (M) for a bunch of masses = $\sum_{i=1}^{n} x_i m_i$

x_i = distance from 0 to particle i

m_i = mass of particle i

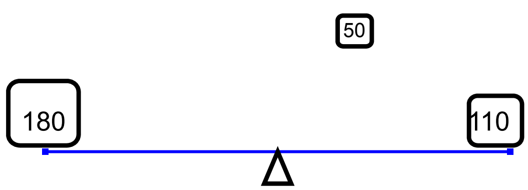


Where does the fulcrum need to be placed to balance? Let's call it $\bar{x}$.



EX 1

John and Mary, weighing 180 lbs and 110 lbs respectively, sit at opposite ends of a 12-ft teeter-totter with the fulcrum in the middle. Where should their 50-lb son sit in order for the board to balance?



33 Moment Center of Mass

For a continuous mass distribution along the line (like on a wire):

$$\bar{x} = \frac{M}{m} = \frac{\int_a^b x\,\delta(x)\,dx}{\int_a^b \delta(x)\,dx}$$

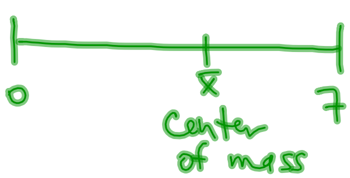
($\delta(x)$ is density fn)

since total mass is $\int_a^b \delta(x)\,dx$

& moment is $\int_a^b \delta(x)\,x\,dx$

EX 2

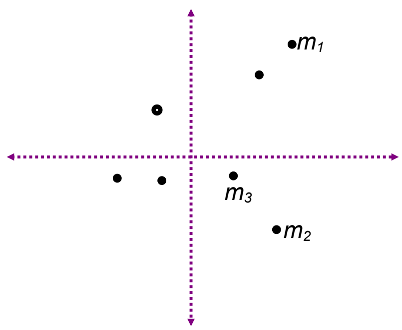
A straight wire 7 units long has density $\delta(x) = 1+x^3$ at a point x units from one end. Find the distance from this end to the center of mass.



33 Moment Center of Mass

Consider a discrete set of 2-d masses.

How do we find the center of mass (the geometric center) $(\overline{x},\overline{y})$?



$\bar{x} = \frac{M_y}{m}$, $\bar{y} = \frac{M_x}{m}$

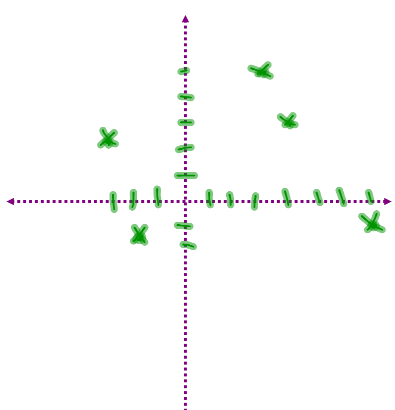
where $M_y = \sum_{i=1}^{n} x_i m_i$

$M_x = \sum_{i=1}^{n} y_i m_i$

total mass $m = \sum_{i=1}^{n} m_i$

EX 3

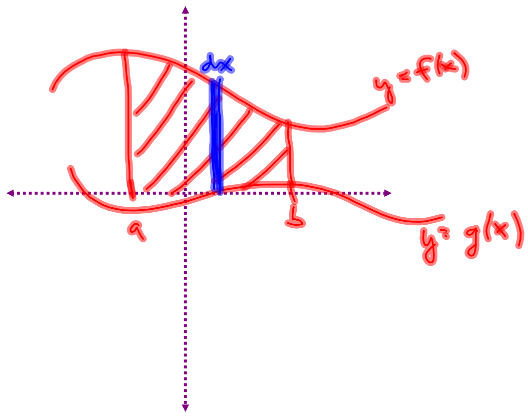
The masses and coordinates of a system of particles are given by the following: 5, (-3,2); 6, (-2,-2); 2, (3,5); 7, (4,3); 1, (7,-1). Find the moments of this system with respect to the coordinate axes and find the center of mass.



33 Moment Center of Mass

Now, consider a continuous 2-d region (a lamina) that has constant (homogeneous) density everywhere. How do we find the center of mass $(\bar{x},\bar{y})$?

It's still true $\bar{x} = \frac{M_y}{m}$, $\bar{y} = \frac{M_x}{m}$



total mass

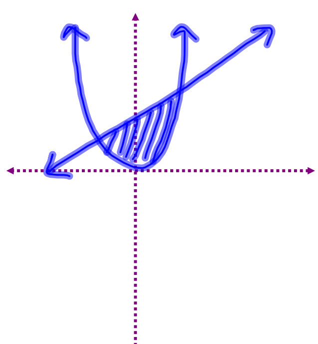
$m = \int_a^b \delta \, (f(x) - g(x))\,dx$

(density · area)

δ = density (per area unit)

EX 4

Find the centroid of the region bounded by $y = x^2$ and $y = x+2$.



33 Moment Center of Mass

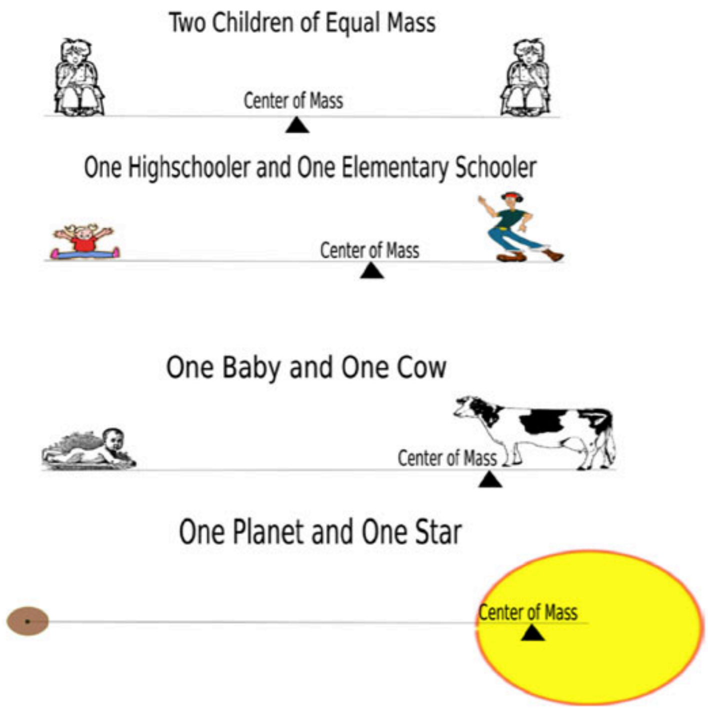


Photo source: Laboratory for Atmospheric and Space Physics, University of Colorado at Boulder