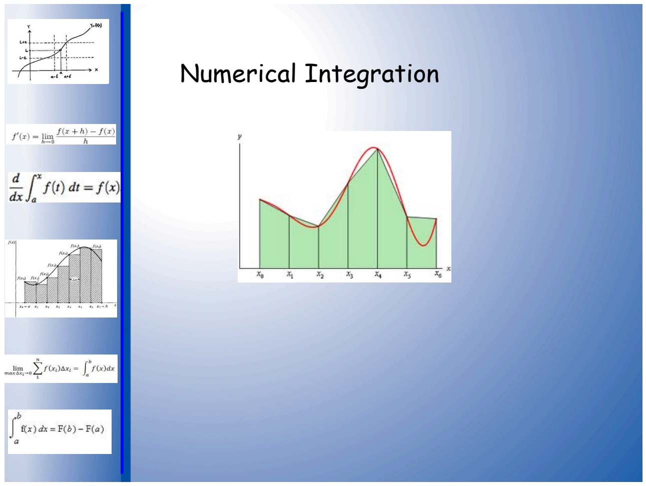
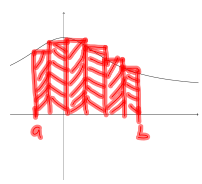


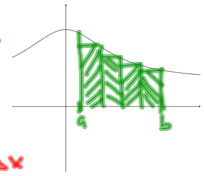
If f(x) is continuous, we are guaranteed that $\int_a^b f(x)\,dx$ exists, but sometimes we cannot evaluate the integral. For these cases, we use numerical methods to approximate the definite integral (area under the curve.)

1. Left Riemann Sum



2. Right Riemann Sum

area of n^{th} rectangle
$= f(x_n)\Delta x_n$



let all $\Delta x_n = \Delta x$.

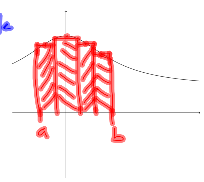
$\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$

$$\int_a^b f(x)\,dx \approx \frac{b-a}{n} \sum_{i=1}^{n} f\left(a + i\left(\frac{b-a}{n}\right)\right)$$

error: $E_n = \frac{-(b-a)^2}{2n} f'(c)$ for some $c \in [a,b]$

3. Midpoint Riemann Sum

area of n^{th} rectangle
$= f\left(\frac{x_{i-1}+x_i}{2}\right)\Delta x$



$\Delta x = \frac{b-a}{n}$

$x_i = a + i\Delta x$

$x_{i-1} = a + (i-1)\Delta x$

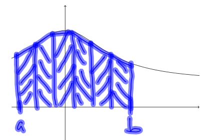
$$\Rightarrow \frac{x_i + x_{i-1}}{2} = \frac{a + i\Delta x + a + (i-1)\Delta x}{2} = \frac{2a + 2i\Delta x - \Delta x}{2}$$

$$= a + i\Delta x - \tfrac{1}{2}\Delta x$$

4. Trapezoidal Rule

area of n^{th} trapezoid

$= \frac{1}{2}\left(f(x_i) + f(x_{i-1})\right)\Delta x$



$\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$

$x_{i-1} = a + (i-1)\Delta x$

$$\int_a^b f(x)\,dx = \frac{1}{2}\left(\frac{b-a}{n}\right)\sum_{i=1}^{n}\left[f(x_{i-1}) + f(x_i)\right]$$

area of right trapezoid:

$A = \frac{1}{2}(y_1 - y_2)h + hy_2$

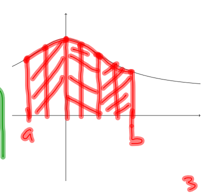
$\Rightarrow A = \frac{1}{2}(y_1 + y_2)h$

5. Simpson's Rule (a.k.a. Parabolic Rule)

$\Delta x = \frac{b-a}{n}$

$x_i = a + i\Delta x$

* n must be even

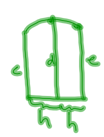


for every two "widths", we connect top 3 pts w/ parabola.

area of one parabolic piece

$= \frac{\Delta x}{3}\left(f(x_i) + 4f(x_{i+1}) + f(x_{i+2})\right)$

Area of parabolic piece:



$A = \frac{h}{3}(c + 4d + e)$

$$\int_a^b f(x)\,dx \approx \left(\frac{b-a}{n}\right)\left(\frac{1}{3}\right)\Big[\left(f(x_0) + 4f(x_1) + f(x_2)\right) + \left(f(x_2) + 4f(x_3) + f(x_4)\right) + \left(f(x_4) + 4f(x_5) + f(x_6)\right) + \ldots + \left(f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right)\Big]$$

$$= \frac{b-a}{3n}\left[f(a) + 4\left(\sum_{i=1}^{n/2} f(a + (2i-1)\Delta x)\right) + 2\left(\sum_{i=1}^{n/2-1} f(a + 2i\Delta x)\right) + f(b)\right]$$

error: $E_n = \frac{-(b-a)^5}{180n^4} f^{(4)}(c)$ for some $c \in [a,b]$

EX 1

Use methods 2, 4 and 5 to approximate this integral. $\int_1^3 \frac{1}{x^3}\,dx$ Let n = 8

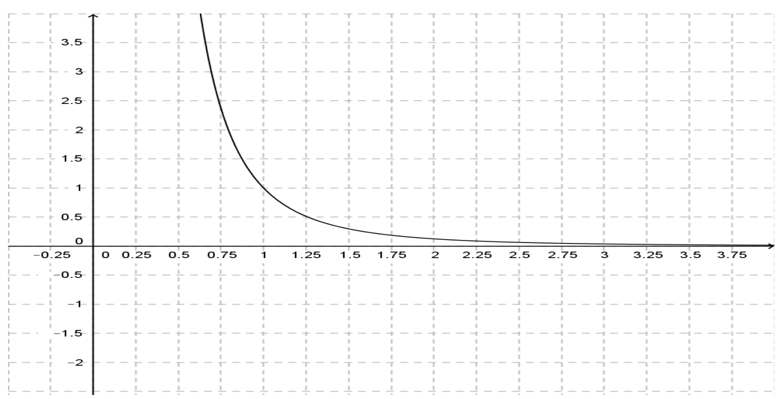
Right Rectangular Method

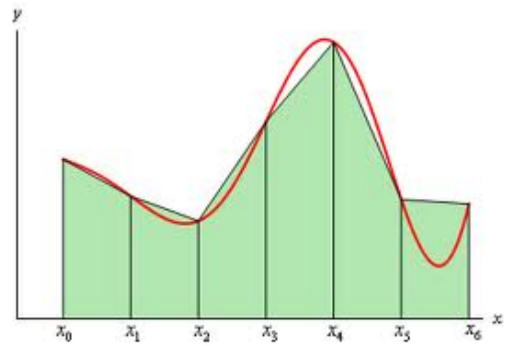
$\int_1^3 \frac{1}{x^3}\,dx$ Let n = 8. Trapezoidal Rule

$\int_1^3 \frac{1}{x^3}\,dx$ Let n = 8. Simpson's Rule

Actual Value

$$\int_1^3 \frac{1}{x^3}\,dx$$




y
x
x_0
x_1
x_2
x_3
x_4
x_5
x_6