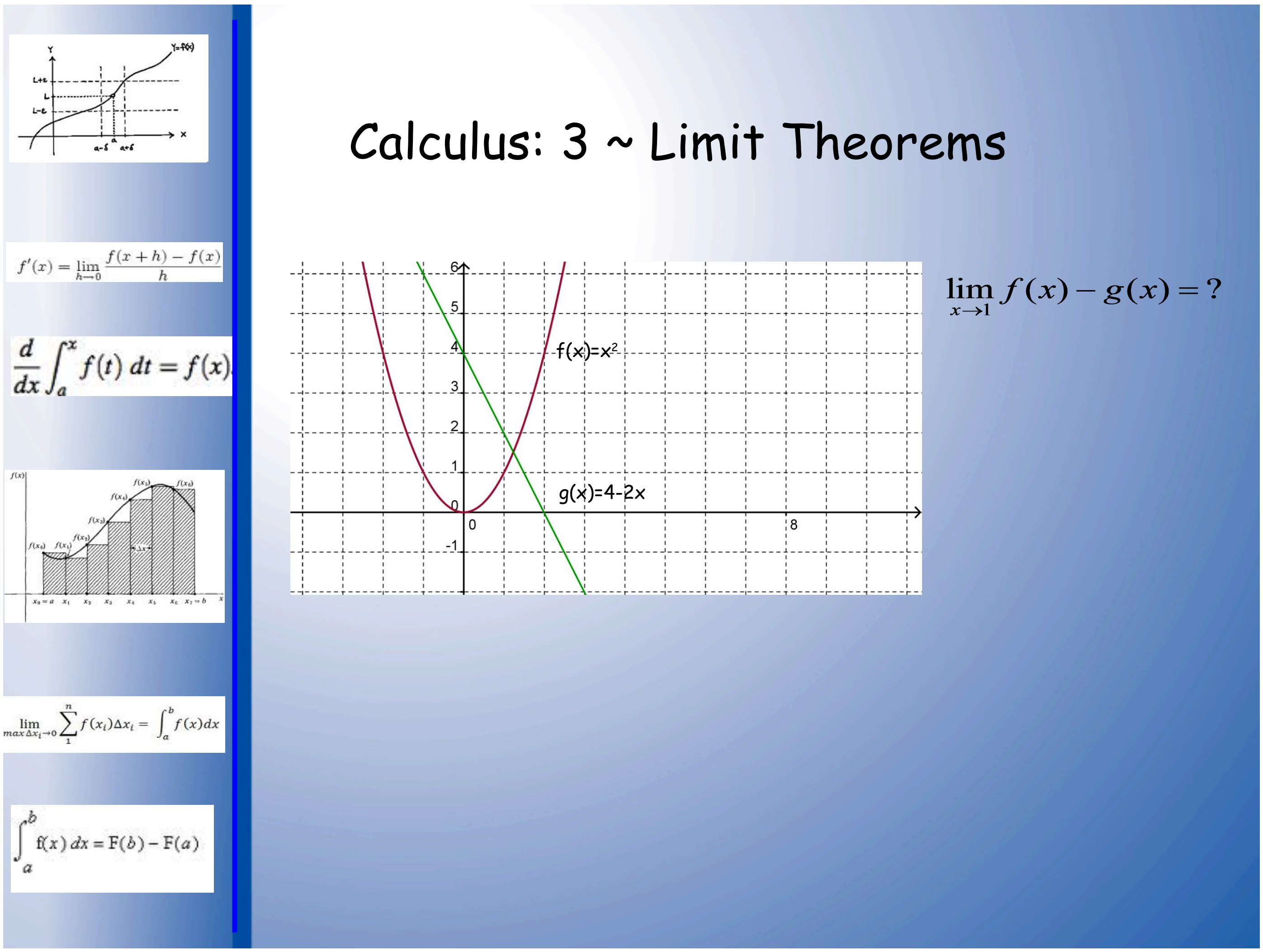


Calculus: 3 ~ Limit Theorems
f(x)=x²
g(x)=4-2x
$\lim_{x\to 1} f(x) - g(x) = ?$
$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$
$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$
$\lim_{max \Delta x_i \to 0} \sum_1^n f(x_i)\Delta x_i = \int_a^b f(x)dx$
$\int_a^b f(x)\,dx = F(b) - F(a)$

Limit Theorems

n is a positive integer.

k is a real number

$f(x)$ & $g(x)$ have limits as $x \to c$

1) $\lim_{x \to c} k = k$ ✓

2) $\lim_{x \to c} x = c$

3) $\lim_{x \to c} [kf(x)] = k \lim_{x \to c} f(x)$

4) $\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$

5) $\lim_{x \to c} [f(x)g(x)] = \lim_{x \to c} f(x) \lim_{x \to c} g(x)$

6) $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}, \quad g(x) \neq 0$

7) $\lim_{x \to c} [f(x)]^n = [\lim_{x \to c} f(x)]^n$

8) $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to c} f(x)}, \quad if \quad \lim_{x \to c} f(x) > 0 \quad when\ n\ is\ even.$

limits can be exchanged in order w/ almost any other operator if the limit exists

EX 1 $\lim_{x\to 2}(4x^2 - 2x + 1)$ $= 4\left(\lim_{x\to 2}(x)\right)^2 - 2\left(\lim_{x\to 2} x\right) + \lim_{x\to 2} 1$

$= 4(2^2) - 2(2) + 1 = 16 - 4 + 1$

$= 13$

EX 2 $\lim_{x\to -3}\frac{\sqrt{x^2-1}}{2x}$

$= \frac{\sqrt{\left(\lim_{x\to -3} x\right)^2 - \lim_{x\to -3} 1}}{2\left(\lim_{x\to -3} x\right)} = \frac{\sqrt{(-3)^2-1}}{2(-3)} = \frac{\sqrt{8}}{-6}$

$= \frac{2\sqrt{2}}{-2(3)} = \frac{-\sqrt{2}}{3}$

EX 3 If $\lim_{x\to a} f(x) = 3$ *and* $\lim_{x\to a} g(x) = -1$,

find $\lim_{x\to a}\frac{2f(x) - 3g(x)}{f(x) + g(x)}$

$= \frac{2\lim_{x\to a} f(x) - 3\lim_{x\to a} g(x)}{\lim_{x\to a} f(x) + \lim_{x\to a} g(x)} = \frac{2(3) - 3(-1)}{3 + -1}$

$= \frac{9}{2}$

Substitution Theorem

If *f(x)* is a polynomial or a rational function, then $\lim_{x\to c} f(x) = f(c)$ assuming *f(c)* is defined.

Ex 4 $\lim_{x\to -1} \frac{3x^2-4x^3+7x-5}{2x^2+3x+4}$ $= \frac{3(-1)^2-4(-1)^3+7(-1)-5}{2(-1)^2+3(-1)+4}$

$= \frac{3+4-7-5}{2-3+4} = \frac{-5}{3}$

Ex 5 $\lim_{x\to 2} \frac{3x^3+4x+1}{x^2-x-2}$ DNE (does not exist)

$\left(\frac{3(2^3)+4(2)+1}{2^2-2-2} = \frac{33}{0} \text{ rase} \right)$

EX 6 $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$ *Hint: rationalize the numerator.*

$\left(\frac{\sqrt{0+1}-1}{0}=\frac{0}{0}\text{ case}\right)$ ($\frac{0}{0}$ case means we have more work to do; simplify algebraically)

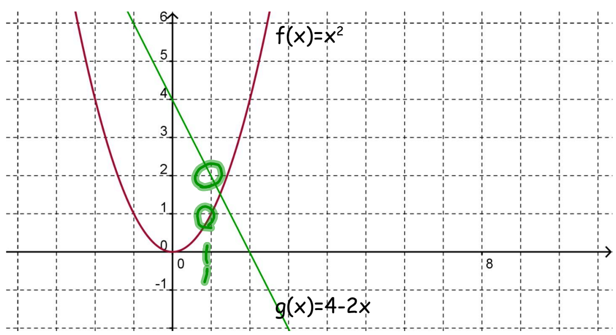
$= \lim_{x\to 0}\frac{(\sqrt{x+1}-1)}{x}\left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right)$

$= \lim_{x\to 0}\frac{(x+1)-1}{x(\sqrt{x+1}+1)} = \lim_{x\to 0}\frac{x}{x(\sqrt{x+1}+1)}$

$= \lim_{x\to 0}\frac{1}{\sqrt{x+1}+1} = \frac{1}{\sqrt{1}+1}$

$= \frac{1}{2}$

3B Limit Theorems



$\lim_{x \to 1} f(x) =$ 1

$\lim_{x \to 1} g(x) =$ 2

$\lim_{x \to 1} f(x) - g(x) = ?$ 1-2=-1