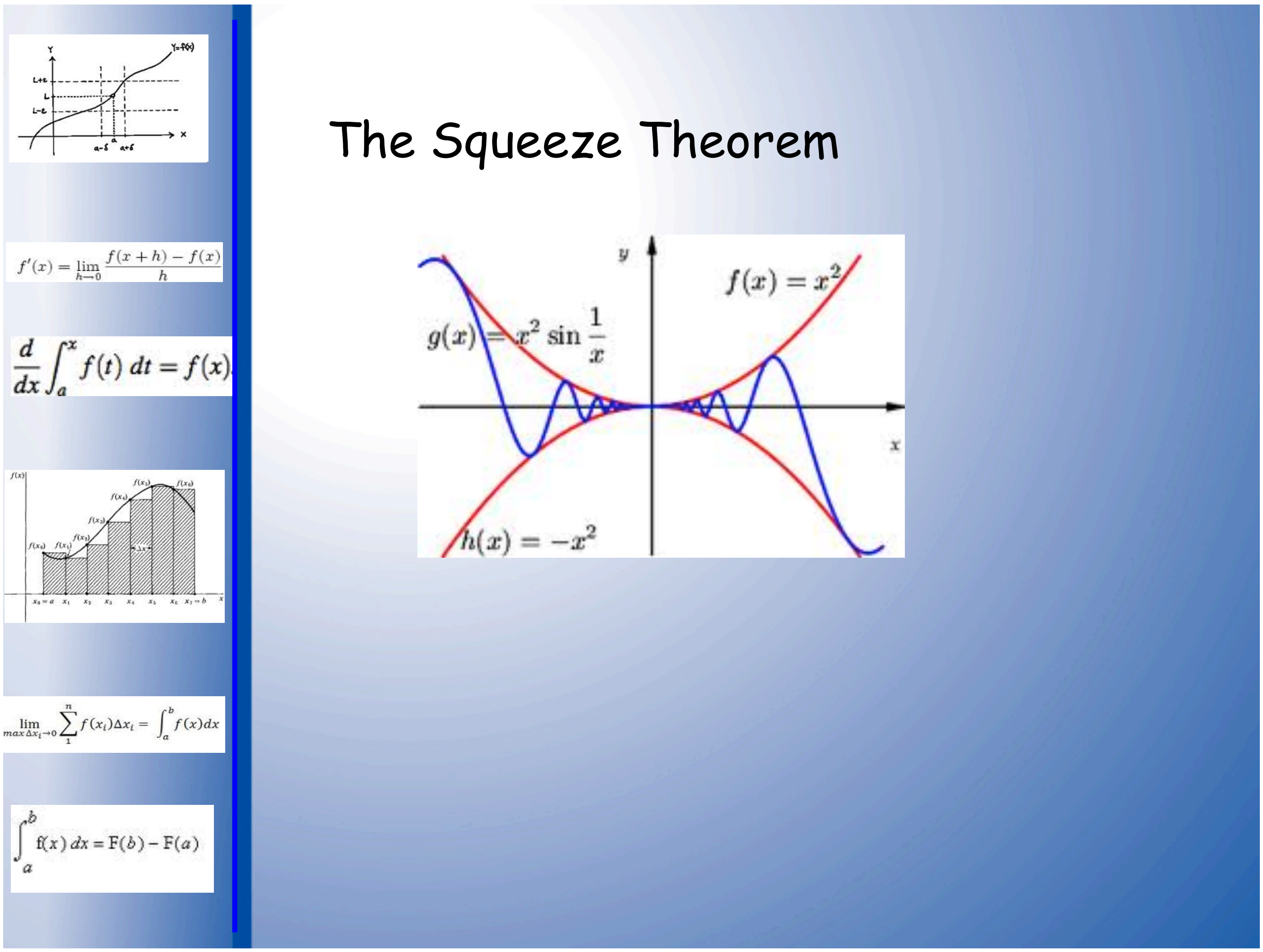


The Squeeze Theorem
$f(x) = x^2$
$g(x) = x^2 \sin \frac{1}{x}$
$h(x) = -x^2$
$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
$\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$
$\lim_{max \Delta x_i \to 0} \sum_1^n f(x_i) \Delta x_i = \int_a^b f(x) dx$
$\int_a^b f(x)\, dx = F(b) - F(a)$

<u>Squeeze Theorem</u>

("squeezing" a fn in between two other fns)

Let *f, g, h* be functions satisfying *f(x) ≤ g(x) ≤ h(x)* for every *x* near *c*, except possibly at *x*=*c*.

If $\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L,$

then $\lim_{x \to c} g(x) = L$

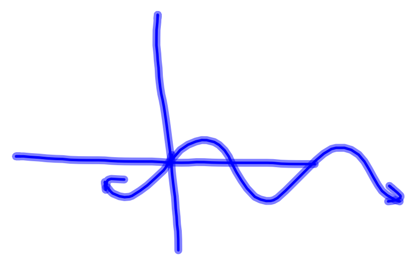
Note: Most frequently used w/ trig fns, like sinx or cosx.

4.5B Squeeze Theorem

Ex 1 Use the squeeze theorem to determine this limit.

$$\lim_{x\to\infty} x^{-1/2}\sin x = \lim_{x\to\infty} \frac{\sin x}{\sqrt{x}}$$

$y = \sin x$



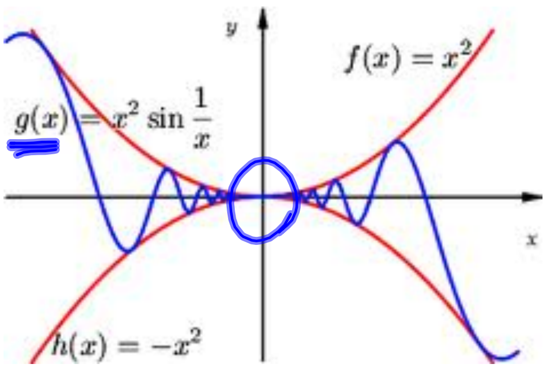
$$\frac{-1}{\sqrt{x}} \le \frac{\sin x}{\sqrt{x}} \le \frac{1}{\sqrt{x}}$$

(because $-1 \le \sin x \le 1$ for all x-values)

$$\underbrace{\lim_{x\to\infty} \frac{-1}{\sqrt{x}}}_{0} \le \lim_{x\to\infty} \frac{\sin x}{\sqrt{x}} \le \underbrace{\lim_{x\to\infty} \frac{1}{\sqrt{x}}}_{0}$$

$$0 \le \lim_{x\to\infty} \frac{\sin x}{\sqrt{x}} \le 0 \implies \boxed{\lim_{x\to\infty} \frac{\sin x}{\sqrt{x}} = 0}$$

logic works if these #s are the same!



blue curve is "squeezed" between the two red curves

$$0 \le \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) \le 0$$

$$\Rightarrow \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0.$$