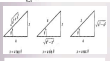


if $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$
or
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$

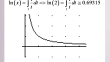
Then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

provided that the latter limit exists.

$f(x) = f(x) + f(x)(x-a) + \frac{f''(x)}{2}(x-a)^2 + \dots$
 $\frac{f'(x)}{2}(x-a)^2 + \frac{f''(x)}{6}(x-a)^3 + \dots$
 $\frac{f''(x)}{6}(x-a)^3 + \dots$



$\sin(x) = \frac{y}{r}$, $\cos(x) = \frac{x}{r}$, $\tan(x) = \frac{y}{x}$



$\int u \, dv = uv - \int v \, du$

Example: $\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$

Trigonometric Integrals

a) $\sin x \cos y = \frac{1}{2} [\sin(x-y) + \sin(x+y)]$

b) $\sin x \sin y = \frac{1}{2} [\cos(x-y) - \cos(x+y)]$

c) $\cos x \cos y = \frac{1}{2} [\cos(x-y) + \cos(x+y)]$

Trigonometric Integrals

Combining u-substitution and the trigonometric identities, we will address three forms of these integrals.

1. $\int \sin^n x \, dx$, $\int \cos^n x \, dx$
2. $\int \sin^m x \cos^n x \, dx$
3. $\int \sin(mx) \cos(nx) \, dx$, $\int \sin(mx) \sin(nx) \, dx$, $\int \cos(mx) \cos(nx) \, dx$

EX 1 $\int \sin^3 x \, dx$

Type 1

If n is odd,

use $\sin^2 x + \cos^2 x = 1$.

If n is even,

use half-angle formulas.

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

EX 2 $\int \cos^4 x \, dx$

Type 1

If n is odd,

use $\sin^2 x + \cos^2 x = 1$.

If n is even,

use half-angle formulas.

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\int \cos^5 x \sin^{-4} x \, dx$$

EX 3 $\int \cos^5 x \sin^{-4} x \, dx$

Type 2

If m or n is odd and positive,

factor out $\sin x$ or $\cos x$

and use $\sin^2 x + \cos^2 x = 1$.

If m and n are even and positive,

use half-angle identities.

EX 4 $\int \cos^2 x \sin^4 x dx$

Type 2

If m or n is odd and positive,
factor out $\sin x$ or $\cos x$

and use $\sin^2 x + \cos^2 x = 1$.

If m and n are even and positive,
use half-angle identities.

EX 5 $\int \sin(4x) \cos(5x) dx$

Type 3

Use product identities:

$$\sin(mx) \cos(nx) = \frac{1}{2} [\sin((m+n)x) + \sin((m-n)x)]$$

$$\sin(mx) \sin(nx) = -\frac{1}{2} [\cos((m+n)x) - \cos((m-n)x)]$$

$$\cos(mx) \cos(nx) = \frac{1}{2} [\cos((m+n)x) + \cos((m-n)x)]$$

EX 6 $\int_{-4}^4 \sin\left(\frac{m\pi x}{4}\right) \sin\left(\frac{n\pi x}{4}\right) dx$