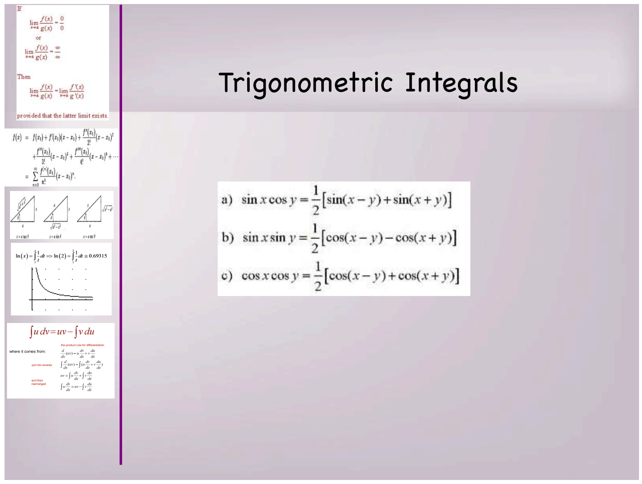

<u>Trigonometric Integrals</u>

Combining u-substitution and the trigonometric identities, we will address three forms of these integrals.

1. $\int \sin^n x\, dx, \quad \int \cos^n x\, dx$
2. $\int \sin^m x \cos^n x\, dx$
3. $\int \sin(mx)\cos(nx)\, dx, \quad \int \sin(mx)\sin(nx)\, dx, \quad \int \cos(mx)\cos(nx)\, dx$

EX 1 $\int \sin^3 x\, dx$

Type 1
If n is odd,
use $\sin^2 x + \cos^2 x = 1$.
If n is even,
use half − angle formulas.
$$\sin^2 x = \frac{1-\cos 2x}{2}$$
$$\cos^2 x = \frac{1+\cos 2x}{2}$$

EX 2 $\int \cos^4 x \, dx$

Type 1
If n is odd,
use $\sin^2 x + \cos^2 x = 1$.
If n is even,
use half − angle formulas.
$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$\int \cos^5 x \sin^{-4} x \, dx$

EX 3 $\int \cos^5 x \sin^{-4} x \, dx$

Type 2
If m or n is odd and positive,
factor out $\sin x$ *or* $\cos x$
and use $\sin^2 x + \cos^2 x = 1$.
If m and n are even and positive,
use half − angle identities.

EX 4 $\int \cos^2 x \sin^4 x \, dx$

Type 2

If m or n is odd and positive, factor out $\sin x$ *or* $\cos x$ *and use* $\sin^2 x + \cos^2 x = 1$.

If m and n are even and positive, use half − angle identities.

EX 5 $\int \sin(4x)\cos(5x)\, dx$

Type 3

Use product identities:

$$\sin(mx)\cos(nx) = \tfrac{1}{2}[\sin((m+n)x) + \sin((m-n)x)]$$

$$\sin(mx)\sin(nx) = -\tfrac{1}{2}[\cos((m+n)x) - \cos((m-n)x)]$$

$$\cos(mx)\cos(nx) = \tfrac{1}{2}[\cos((m+n)x) + \cos((m-n)x)]$$

EX 6 $\int_{-4}^{4} \sin\left(\frac{m\pi x}{4}\right)\sin\left(\frac{n\pi x}{4}\right) dx$