

if $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{0}{0}$
or
 $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$

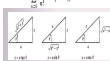
Then $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$

provided that the latter limit exists.

$f(x) = f(x) + f(x) - f(x) = \frac{P(x)}{Q(x)} - \frac{P(x)}{Q(x)}$

$\frac{P(x)}{Q(x)} = \frac{P(x)}{Q(x)} + \frac{P(x)}{Q(x)} - \frac{P(x)}{Q(x)}$

$\frac{P(x)}{Q(x)} = \frac{P(x)}{Q(x)} + \frac{P(x)}{Q(x)} - \frac{P(x)}{Q(x)}$



$\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$

$\frac{1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$

$1 = A(x+2) + B(x-2)$

$1 = Ax + 2A + Bx - 2B$

$1 = (A+B)x + (2A-2B)$

$0 = A+B$
 $1 = 2A-2B$

$A = -B$
 $1 = 2A - 2(-A) = 4A$
 $A = \frac{1}{4}$
 $B = -\frac{1}{4}$

$\frac{1}{x^2-4} = \frac{1/4}{x-2} - \frac{1/4}{x+2}$

$\int \frac{1}{x^2-4} dx = \frac{1}{4} \int \frac{1}{x-2} dx - \frac{1}{4} \int \frac{1}{x+2} dx$

$= \frac{1}{4} \ln|x-2| - \frac{1}{4} \ln|x+2| + C$

Integration of Rational Functions Using Partial Fraction Decomposition

$$\frac{a}{x-2} + \frac{b}{x+2} = \frac{4}{(x-2)(x+2)}$$

$$\frac{a(x+2)}{(x-2)(x+2)} + \frac{b(x-2)}{(x+2)(x-2)} = \frac{4}{(x-2)(x+2)}$$

$$\frac{(a+b)x + 2(a-b)}{(x-2)(x+2)} = \frac{4}{(x-2)(x+2)}$$

Partial Fraction Decomposition

A rational function is the quotient of two polynomials.

A proper rational function is the quotient of two polynomials where the numerator has a lower degree than the denominator.

Review of partial fraction decomposition (pfd)

EX 1 Rewrite this as a sum/difference of two fractions.

$$\frac{x-7}{x^2-x-12}$$

EX 2 $\int \frac{4x^2 - 6x + 2}{x^2(x-1)(x+3)} dx$

EX 3 $\int \frac{33x^2 - 7x + 70}{(3x-2)(x^2+4)} dx$

$$\text{EX 4 } \int \frac{\cos x}{\sin^4 x - 16} dx$$

$$\text{EX 5 } \int \frac{x^6 - 7x^4 + 11x^3 - 13x^2 + x - 6}{x^3 - 2x^2} dx$$