

Sequence and Series Review

If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{0}{0}$
 or
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}$
 Then
 $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$
 provided that the latter limit exists.

$f(x) = f(x_1) + f'(x_1)(x-x_1) + \frac{f''(x_1)}{2!}(x-x_1)^2 + \frac{f'''(x_1)}{3!}(x-x_1)^3 + \dots$
 $= \sum_{k=0}^{\infty} \frac{f^{(k)}(x_1)}{k!}(x-x_1)^k$

$\ln(x) = \int_1^x \frac{1}{t} dt \Rightarrow \ln(2) = \int_1^2 \frac{1}{t} dt \approx 0.69315$

$\int u dv = uv - \int v du$

where it comes from:
 $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
 put into reverse: $\int \frac{d}{dx}(uv) = \int u \frac{dv}{dx} + \int v \frac{du}{dx}$
 and then rearrange: $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

$2, 4, 6, 8, 10, \dots$	$2, 4, 8, 16, 32, \dots$
$a_n = a_1 + (n-1)d$	$a_n = a_1 r^{(n-1)}$
$\sum_{k=1}^6 a_1 + (k-1)d$	$\sum_{k=1}^6 a_1 r^{k-1}$
$S_n = \frac{n}{2}(a_1 + a_n)$	$S_n = a_1 \frac{1-r^n}{1-r}$

A **sequence** $\{a_n\}$ is a function such that the domain is the set of positive integers and the range is a set of real numbers.

Write five terms for each of these sequences:

$$a_n = \frac{n}{2n+1}$$

$$a_n = \frac{(-2)^n}{n!}$$

A **series** is the sum of a sequence. $\sum_{k=1}^n a_k$

A **partial sum** is the sum of the first n terms.

An **infinite sum** is the sum from k= 1 to ∞ .

Find these partial sums:

$$\sum_{k=0}^3 \frac{(-2)^k}{k!}$$

$$\sum_{k=1}^5 \frac{k}{2k+1}$$

Arithmetic Sequence, Series

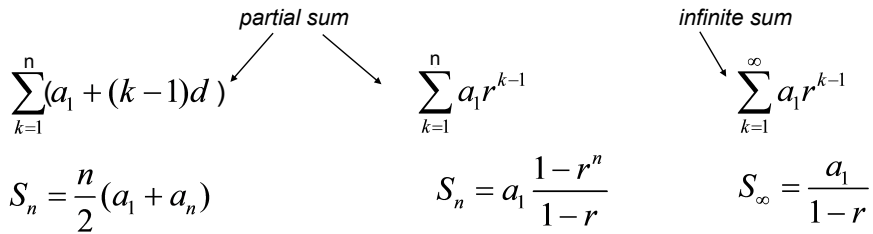
d = common difference

$$a_n = a_1 + (n-1)d$$

Geometric Sequence, Series

r = common ratio

$$a_n = a_1 r^{(n-1)}$$



Determine the sum for each of these:

$$\sum_{k=1}^{50} 2k - 3$$

$$\sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^k$$

Common Elements of Sequences/Series:

Odd numbers

Even numbers

Factorials

Alternating signs

Powers of 2

Arithmetic, Geometric or Neither?

n^{th} term 20^{th} term

$a_n \rightarrow 0?$

$\sum_{k=1}^{\infty} a_k \rightarrow \text{some value?}$

a) $1, 1, 2, 3, 5, 8, 13, \dots$

b) $a_1 = 2, a_{n+1} = \frac{a_n}{n}$

c) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

d) $.9, .09, .009, .0009, \dots$

Write a formula for the n^{th} term of these sequences.

e) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \dots$

f) $\frac{1}{2}, \frac{1}{4}, \frac{1}{16}, \frac{1}{256}, \dots$

g) $\frac{-2}{1}, \frac{8}{2}, \frac{-26}{6}, \frac{80}{24}, \frac{-242}{120}$