

## Positive Series: Integral Test

If  $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \frac{0}{0}$  or  $\frac{\infty}{\infty}$  then  $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{f'(n)}{g'(n)}$

Then  $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{f'(n)}{g'(n)}$

provided that the latter limit exists.

$f(n) = f(n) + f(n)(n-1) + \frac{f''(n)}{2!}(n-1)^2 + \dots + \frac{f^{(k)}(n)}{k!}(n-1)^k + \dots$

$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \lim_{n \rightarrow \infty} \frac{f'(n)}{g'(n)}$

$\int_1^{\infty} f(x) dx = \lim_{b \rightarrow \infty} \int_1^b f(x) dx = \lim_{b \rightarrow \infty} [F(b) - F(1)] = \lim_{b \rightarrow \infty} F(b) - F(1)$

$\int u dv = uv - \int v du$

**Example:**

Determine whether the harmonic series  $\sum_{n=1}^{\infty} \frac{1}{n}$  converges or diverges.

**Solution:**

Using the integral test for convergence:

$$\int_1^{\infty} \frac{dx}{x} = \lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x} = \lim_{b \rightarrow \infty} \ln(b) = \infty$$

$\therefore$  Series diverges

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### Bounded Sum Test

A series  $\sum a_i$  of nonnegative terms converges if and only if its partial sums are bounded above.

EX 1 Does  $\sum_{k=1}^{\infty} \frac{|\sin k|}{(k+1)!}$  converge?

### Integral Test

If  $f(x)$  is continuous, positive and nonincreasing on  $[N, \infty)$

and  $a_k = f(k)$  for all positive integers,  $k$ , then

$\sum_{n=N}^{\infty} a_n$  converges if and only if  $\int_N^{\infty} f(x)dx$  converges.

EX 2 Does  $\sum_{k=1}^{\infty} \frac{5k^2}{1+k^3}$  converge or diverge?

*p*-series test

$\sum_{k=1}^{\infty} \frac{1}{k^p}$  is called a *p*-series. It converges if  $p > 1$  and diverges if  $p \leq 1$ .

EX 3 Does  $\sum_{k=1}^{\infty} \frac{1}{k^3}$  converge or diverge?

EX 4 Estimate the error made by approximating the series by the sum of the first five terms.

$$E_n = \sum_{k=n}^{\infty} \frac{1}{k\sqrt{k}} \qquad S_n = \sum_{k=1}^n \frac{1}{k\sqrt{k}}$$