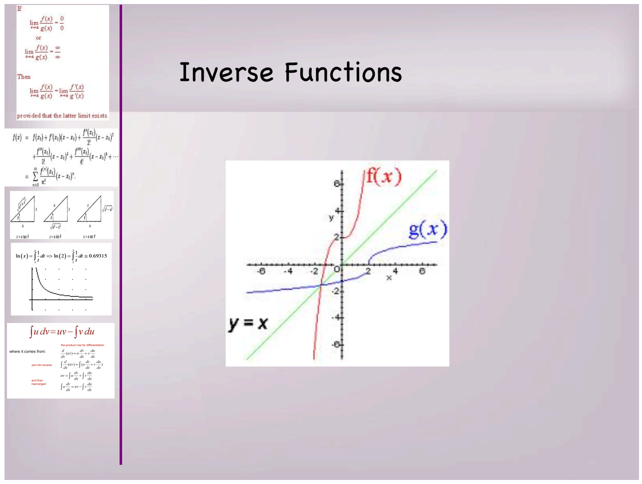


Inverse Functions

If $f(x)$ and $f^{-1}(x)$ are inverse functions:

* $f(x)$ must be one-to-one,
  I.e. The inverse exists when we can get back to an x given a y.
  The horizontal line test may be used.

* If (a,b) is on $f(x)$,then (b,a) is on $f^{-1}(x)$.

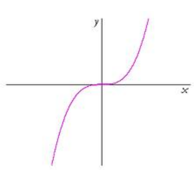
* $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

* *The domain of $f(x)$ becomes the range of $f^{-1}(x)$*
* *The range of $f(x)$ becomes the domain of $f^{-1}(x)$*

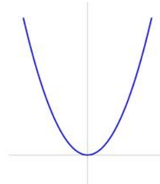
Note: $f^{-1}(x)$ is not the reciprocal, $\dfrac{1}{f^{-1}(x)}$

Let's look at two functions:

$f(x) = x^3$



$f(x) = x^2$



If we don't have a graph, how can we algebraically test if a function has an inverse?

Theorem A If f is strictly monotonic on its domain, then f has an inverse.

EX 1 Show that this function has an inverse, but do not find it.

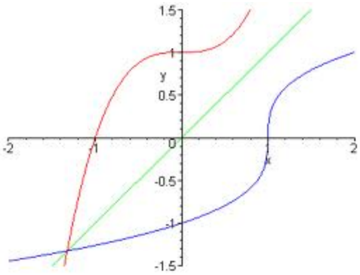
$$f(x) = 3x^7 + 4x^3 + x - 3$$

EX 2 Explore whether this function has an inverse. If not, can we restrict the domain so it does? If so, find $f^{-1}(x)$.

$$f(x) = x^2 - 4$$

EX 3 Find $f^{-1}(x)$ for this function and check your work.

$$y = \frac{2x-1}{3+5x}$$



The graph of $f^{-1}(x)$ is $f(x)$ reflected across the line $y = x$.

Notice the slope at (d,c) and the slope at (c,d).

$$\left(f^{-1}\right)'(d) = \frac{1}{f'(c)}$$

<u>Theorem B: Inverse Function Theorem</u>

If f is differentiable, strictly monotonic on an interval and $f'(x) \neq 0$ at some x on the interval,

then $f^{-1}(x)$ is differentiable at a corresponding point in the range of f and $\left(f^{-1}\right)'(y) = \dfrac{1}{f'(x)}$.

$$\frac{dx}{dy} = \frac{1}{dy/dx}$$

EX 4 Find $(f^{-1})'(2)$ using theorem B. $f(x) = x^5 + 5x - 4$