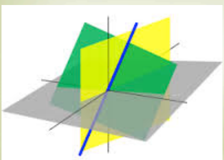
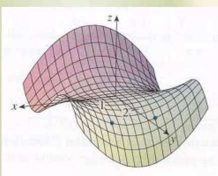
Functions of Two or More Variables



$$f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

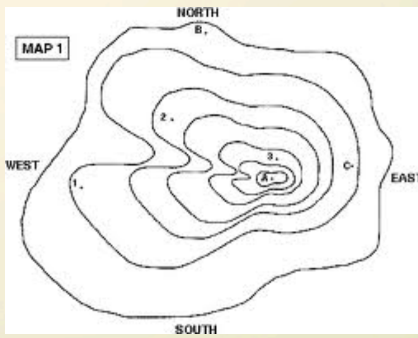
$$f_y = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

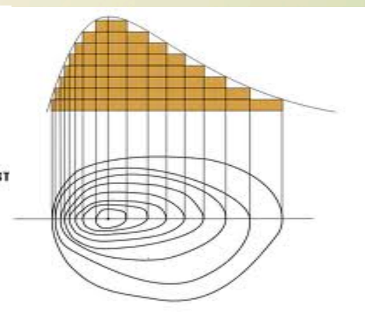


$$\int_0^1 \int_0^{2y} xy\,dx\,dy = \int_0^1 \left[\frac{x^2}{2} y\right]_{x=0}^{x=2y} dy$$

$$= \int_0^1 \frac{(2y)^2}{2} y\,dy = \int_0^1 2y^3\,dy$$

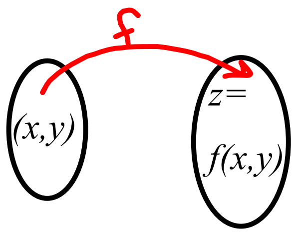
$$= \left[\frac{y^4}{2}\right]_{y=0}^{y=1} = \frac{1}{2}$$





A <u>real-valued function of 2 variables</u> takes two real input values and returns one real output value. (scalar valued fn) (note: this is another type of fn.)

e.g. $f(x,y)=x^2+3y^2$ or $g(x,y)=\sqrt{xy}+2x^3$.



independent variables ⇒ x and y (inputs)

dependent variable ⇒ z (output)

domain⇒ set of allowable inputs (ordered pairs (x,y))

range⇒ set of outputs

EX 1 $f(x,y)=\dfrac{y}{x}+xy$, find

a) $f(1,2)=\dfrac{2}{1}+1(2)=4$ (this surface goes thru pt (1,2,4) in 3d.)

b) $f(a,a)=\dfrac{a}{a}+a(a)=1+a^2$ " " $(a,a,1+a^2)$

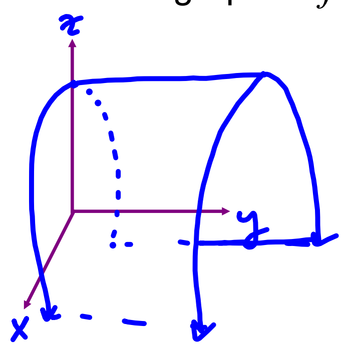
c) $f(\frac{1}{x},x^2)=\dfrac{x^2}{1/x}+\dfrac{1}{x}(x^2)=x^3+x$

d) What is the domain of *f*? $x\neq 0$

domain: $x\in(-\infty,0)\cup(0,\infty)$
$y\in(-\infty,\infty)$

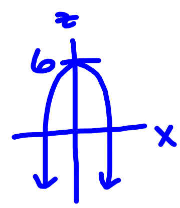
The <u>graph of a function of *2* variables</u> is a *3*D surface (usually). Since it is a function, then to each output, *z,* there can only be one *(x,y)* from the domain. Graphically, this means that each line perpendicular to the *xy*-plane intersects the surface in at most one point.

EX 2 Sketch the graph of $f(x,y) = 6 - x^2$.

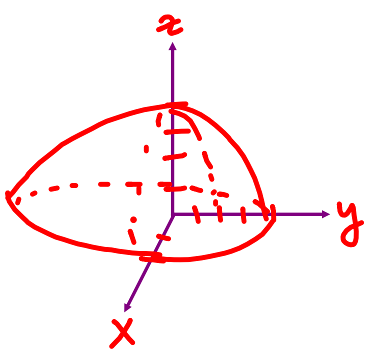


$z = 6 - x^2$

(cylinder along y-axis)



EX 3 Sketch the graph of $f(x,y) = \sqrt{16 - 4x^2 - y^2}$.



$z = \sqrt{16 - 4x^2 - y^2}$

$4x^2 + y^2 + z^2 = 16$

$\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{16} = 1$

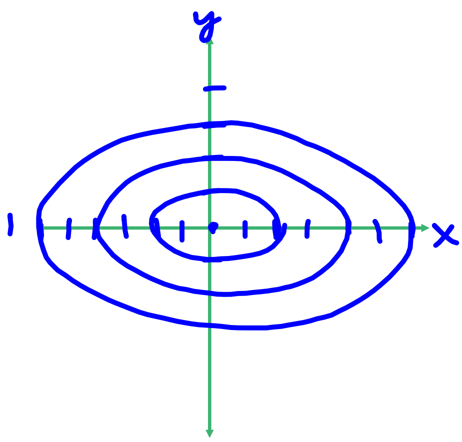
(ellipsoid)

(only top half)

Level Curves ⇒ Projection of intersecting curves (with surface and planes $z = c$, c is real) onto the xy-plane.

Contour Map ⇒ a collection of level curves.

EX 4 Sketch level curves at $z = -1, 0, 1, 4, 9$ for $z = \frac{1}{4}x^2+y^2$.



$z=-1$: $-1 = \frac{1}{4}x^2 + y^2$ N.S. (no level curve)

$z=0$: $0 = \frac{1}{4}x^2 + y^2$

⇒ trivial soln $(0,0)$

$z=1$: $1 = \frac{x^2}{4} + y^2$

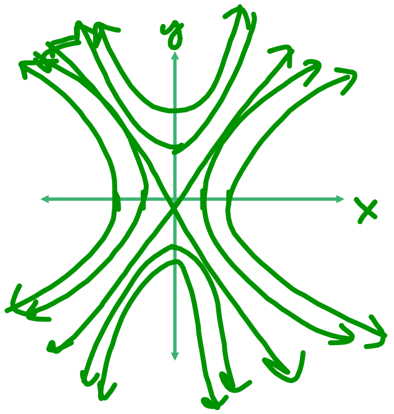
ellipse

$z=4$: $4 = \frac{x^2}{4} + y^2$

$1 = \frac{x^2}{16} + \frac{y^2}{4}$ ellipse

$z=9$: $1 = \frac{x^2}{36} + \frac{y^2}{9}$

EX 5 Sketch level curves at $z = -4, -1, 0, 1, 4$ for $z = y^2 - x^2$. (surface: hyperbolic paraboloid)



$z=-4$: $-4 = y^2 - x^2$

$1 = \frac{x^2}{4} - \frac{y^2}{4}$ hyperbola

$z=-1$: $1 = x^2 - y^2$

$z=0$: $0 = y^2 - x^2 \iff x = \pm y$

$z=1$: $1 = y^2 - x^2$

$z=4$: $1 = \frac{y^2}{4} - \frac{x^2}{4}$

EX 6 Find the domain for $f(x, y, z) = \sqrt{x^2 + y^2 - z^2 - 9}$.

(lives in 4d space)

$w = f(x,y,z)$

3 indep. input variables (x,y,z)

1 output var.

domain: $x^2+y^2-z^2-9 \geq 0$

$x^2+y^2-z^2 \geq 9$

(hyperboloid of one sheet)