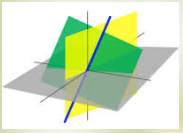
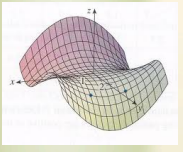
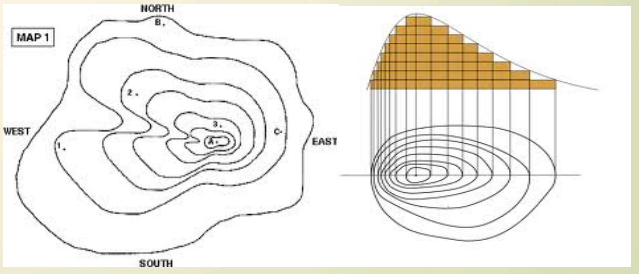


$$f_x = \frac{\partial f}{\partial x} = \lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$$
$$f_y = \frac{\partial f}{\partial y} = \lim_{h\to 0} \frac{f(x,y+h)-f(x,y)}{h}$$



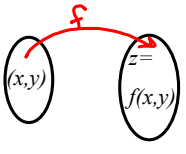
$$\int_0^1 \int_0^{2y} xy\,dx\,dy = \int_0^1 \left[\frac{x^2}{2}y\right]_{x=0}^{x=2y} dy$$
$$= \int_0^1 \frac{(2y)^2}{2} y\,dy = \int_0^1 2y^3\,dy$$
$$= \left[\frac{y^4}{2}\right]_{y=0}^{y=1} = \frac{1}{2}$$

Functions of Two or More Variables



A real-valued function of 2 variables takes two real input values and returns one real output value.

e.g. $f(x,y) = x^2 + 3y^2$ or $g(x,y) = \sqrt{xy} + 2x^3$.



independent variables ⇒

dependent variable ⇒

domain⇒

range⇒

EX 1 $f(x,y) = \frac{y}{x} + xy$, find

a) $f(1,2)$

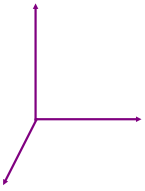
b) $f(a,a)$

c) $f(\frac{1}{x}, x^2)$

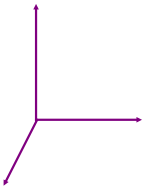
d) What is the domain of f?

The graph of a function of *2* variables is a *3*D surface (usually). Since it is a function, then to each output, *z,* there can only be one *(x,y)* from the domain. Graphically, this means that each line perpendicular to the *xy*-plane intersects the surface in at most one point.

EX 2 Sketch the graph of $f(x,y) = 6 - x^2$.



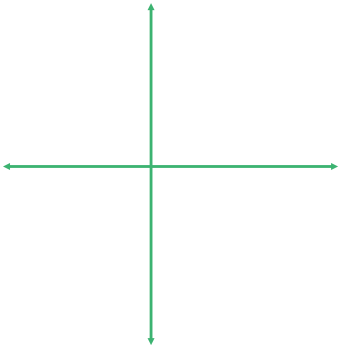
EX 3 Sketch the graph of $f(x,y) = \sqrt{16 - 4x^2 - y^2}$.



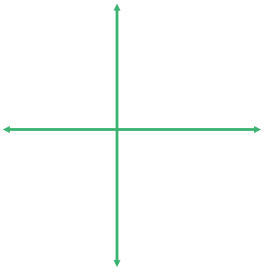
Level Curves ⇒ Projection of intersecting curves (with surface and planes $z = c$, c is real) onto the *xy*-plane.

Contour Map ⇒ a collection of level curves.

EX 4 Sketch level curves at $z = -1, 0, 1, 4, 9$ for $z = \frac{1}{4}x^2 + y^2$.



EX 5 Sketch level curves at $z = -4, -1, 0, 1, 4$ for $z = y^2 - x^2$.



EX 6 Find the domain for $f(x, y, z) = \sqrt{x^2 + y^2 - z^2 - 9}$.