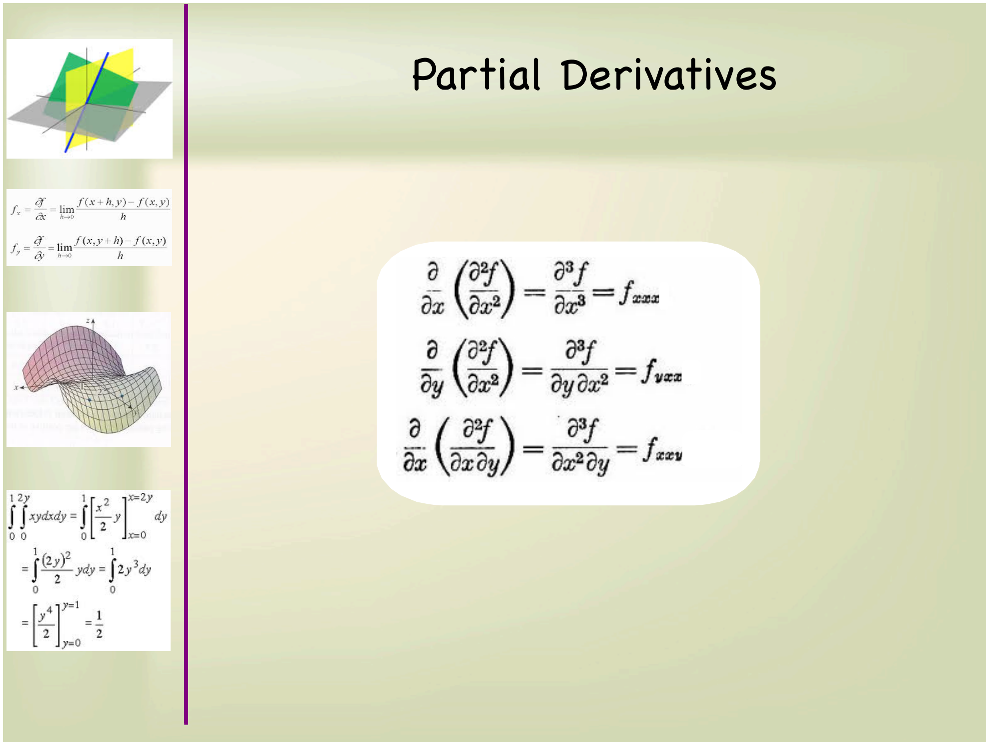
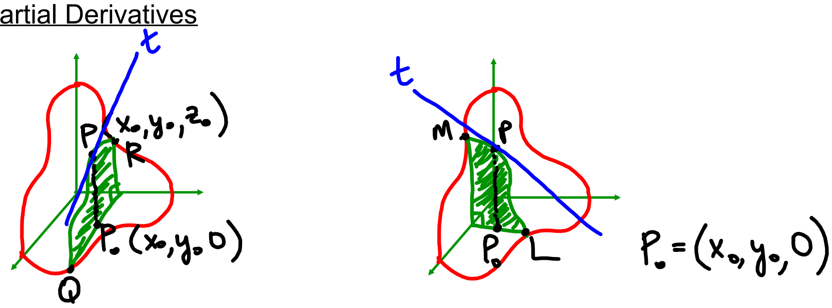


Partial Derivatives



a **b**

Consider the same surface cut by two different planes.

In **a** it is cut by $y = y_0$,

in **b** it is cut by $x = x_0$.

The curve of intersection in **a** goes through plane RPQ and in **b** through plane MPL.

Each of those curves has a tangent line associated with it at point P.

Each tangent line has a steepness associated with it and that should make us think about what?

Since our function is now a function of two variables (rather than one), we can only take the partial derivative with respect to one of the variables.

$$f_x(x_0, y_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

$$f_y(x_0, y_0) = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

EX 1 Find $f_x(0,3)$ and $f_y(0,3)$ if $f(x,y) = 3x^2y^2 + 4y^3 - 5$.

Notation

If $z = f(x,y)$, then

$$f_x(x,y) = \frac{\partial z}{\partial x} = \frac{\partial f(x,y)}{\partial x}$$ partial derivative of f with respect to x

$$f_y(x,y) = \frac{\partial z}{\partial y} = \frac{\partial f(x,y)}{\partial y}$$ partial derivative of f with respect to y

EX 2 If $z = x^2y + \cos(xy) - 2$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

EX 3 Find the 'slope' of the tangent line to the curve of intersection of this surface $3z = \sqrt{36 - 9x^2 - 4y^2}$

and the plane $x = 1$

at the point $(1, -2, \sqrt{11}/3)$.

The 'slope' here refers to the change in z over the change in y.

EX 4 The temperature in degrees celsius on a metal plate in the xy-plane is given by $T(x,y) = 4 + 2x^2 + y^3$. What is the rate of change of temperature with respect to distance (in feet) if we start moving from $(3,2)$ in the direction of the y-axis?

Higher Order Partial Derivatives

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} \qquad f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2}$$

$$f_{xy} = \left(f_x\right)_y = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y\,\partial x} \qquad f_{yx} = \left(f_y\right)_x = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x\,\partial y}$$

EX 5 Find all four second partial derivatives for $f(x,y) = (x^3 + y^2)^5$.

EX 6 Find all four second partial derivatives for $f(x,y) = \tan^{-1}(xy)$.

EX 7 For $f(x, y, z) = xy^2 - \frac{2x}{yz} + 3z^3x$, find f_x, f_y, f_z, f_{xz} *and* f_{yy}.