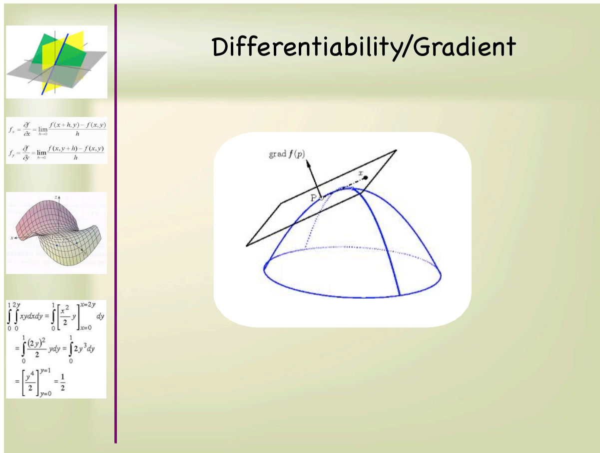


Differentiability

For a function of one variable, the derivative gives us the slope of the tangent line, and a function of one variable is differentiable if the derivative exists. For a function of two variables, the function is differentiable at a point if it has a tangent plane at that point. But existence of the first partial derivatives is not quite enough, unlike the one-variable case.

Theorem

If $f(x,y)$ has continuous partial derivatives $f_x(x,y)$ and $f_y(x,y)$ on a disk D whose interior contains (a,b), then $f(x,y)$ is differentiable at (a,b).

Theorem

If f is differentiable at (a,b), then f is continuous at (a,b).

Gradient of f

$$\nabla f(p) = \nabla f(a,b) = \left\langle f_x(a,b), f_y(a,b) \right\rangle = f_x(a,b)\hat{i} + f_y(a,b)\hat{j}$$

for a function, $z = f(x,y)$.
(Note: This gradient lives in 2-D space, but it is the gradient of a function whose graph is 3-D.)

Properties of Gradient Operator
p is the input point (a,b).

$$\nabla[f(p) + g(p)] = \nabla f(p) + \nabla g(p)$$
$$\nabla[\alpha f(p)] = \alpha \nabla f(p), \alpha \in \Re$$
$$\nabla[f(p)g(p)] = f(p)\nabla g(p) + \nabla f(p) g(p)$$

EX 1 Find the gradient of f.

a) $f(x,y) = x^3y - y^3$

b) $f(x,y) = \sin^3(x^2y)$

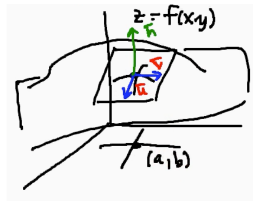
c) $f(x,y,z) = xz \ln(x+y+z)$

Tangent Plane

Curves in 2-D

Remember the equation of the tangent line to a 2-D curve.

Surfaces in 3-D



EX 2 For $f(x,y) = x^3y+3xy^2$, find the equation of the tangent plane at $(a,b) = (2,-2)$.

Ex 3 Find the equation of the tangent "hyperplane" to $f(x,y,z)$ at the point (a,b,c).

$f(x,y,z) = xyz+x^2 \quad (a,b,c) = (2,0,-3)$

Ex 4 Find all domain points (x,y) at which the tangent plane to the graph of $z = x^3$ is horizontal.