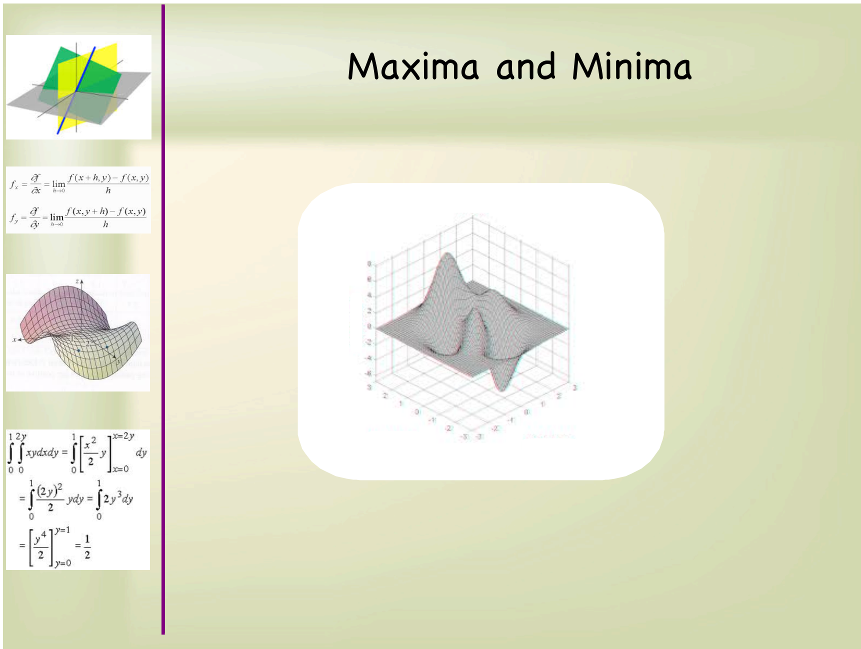

Recall from Calculus I:

1) Critical points (where $f'(x) = 0$ or DNE) are the candidates for where local min and max points can occur.

2) You can use the Second Derivative Test (SDT) to test whether a given critical point is a local min or max. SDT is not always conclusive.

3) Global max and min of a function on an interval can occur at a critical point in the interior of the interval or at the endpoints of the interval.

Extreme Values

1) f has a global maximum at a point (a,b) if $f(a,b) \geq f(x,y)$ for all (x,y) in the domain of f. f has a local maximum at a point (a,b) if $f(a,b)) \geq f(x,y)$ for all (x,y) near (a,b).

2) f has a global minimum at a point (a,b) if $f(a,b) \leq f(x,y)$ for all (x,y) in the domain of f. f has a local minimum at a point (a,b) if $f(a,b)) \leq f(x,y)$ for all (x,y) near (a,b).

Theorem (Critical Point)

Let f be defined on a set S containing (a,b). If $f(a,b)$ is an extreme value (max or min), then (a,b) must be a critical point, i.e. either (a,b) is

a) a boundary point of S

b) a stationary point of S (where $\nabla f(a,b) = \vec{0}$, i.e. the tangent plane is horizontal)

c) a singular point of S (where f is not differentiable).

Fact: Critical points are candidate points for both global and local extrema.

Theorem (Max-Min Existence)

If f is continuous on a closed, bounded set S, then f attains both a global max value and a global min value there.

Second Partials Test Theorem

Suppose $f(x,y)$ has continuous second partial derivatives in a neighborhood of (a,b) and $\nabla f(a,b) = \vec{0}$.

Let $D = D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - f_{xy}^2(a,b)$

then

1) If $D>0$ and $f_{xx}(a,b)<0$, then $f(a,b)$ is a local max.
2) If $D>0$ and $f_{xx}(a,b)>0$, then $f(a,b)$ is a local min.
3) If $D<0$, then $f(a,b)$ is not an extreme value.
   ((a,b) is a saddle point.)
4) If $D = 0$ the test is inconclusive.

EX 1 For $f(x,y) = xy^2 - 6x^2 - 3y^2$, find all critical points, indicating whether each is a local min, a local max or saddle point.

EX 2 Find the global max and min values for

$f(x,y) = x^2 - y^2 - 1$ on

$S = \{(x,y) | x^2 + y^2 \leq 1\}$

EX 3 Find the points where the global max and min occur for

$f(x,y) = x^2 + y^2$ on $S = \{(x,y) | x \in [-1,3], y \in [-1,4]\}$.

EX 4 Find the 3-D vector of length 9 with the largest possible sum of its components.