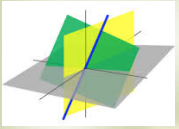
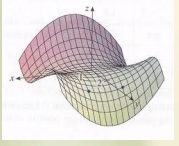


$$f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

$$f_y = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}$$

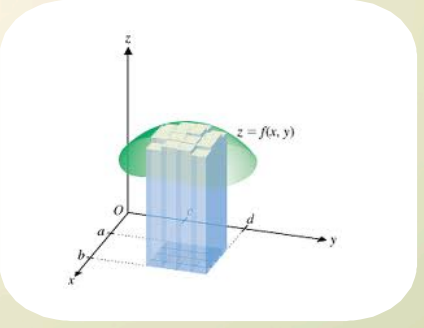


$$\int_0^1 \int_0^{2y} xy\,dx\,dy = \int_0^1 \left[\frac{x^2}{2} y\right]_{x=0}^{x=2y} dy$$

$$= \int_0^1 \frac{(2y)^2}{2} y\,dy = \int_0^1 2y^3\,dy$$

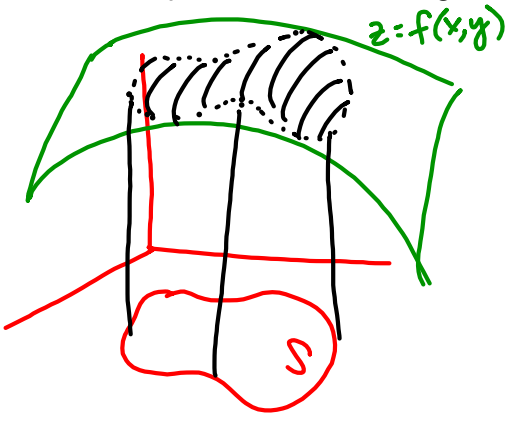
$$= \left[\frac{y^4}{2}\right]_{y=0}^{y=1} = \frac{1}{2}$$

Double Integrals Over Non-rectangular Regions



Double Integrals over Non-rectangular Regions

What if the region we're integrating over is not a rectangle, but a simple, closed curve region instead?

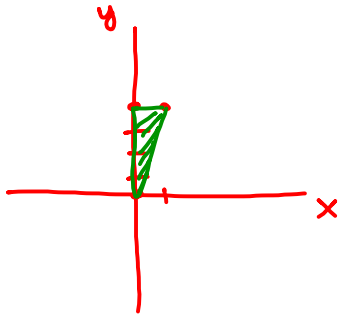


$$V = \iint_S f(x,y)\,dA$$

$$= \int_a^b \int_{\phi_1(x)}^{\phi_2(x)} f(x,y)\,dy\,dx$$

$$= \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

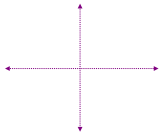
EX 1 Find $\iint_S 5\,dA$ where S is the triangle with vertices at $(0,0)$, $(0,4)$, and $(1,4)$.



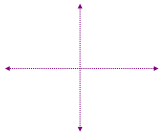
EX 2 Evaluate $\iint_S x\,dA$ where S is the region between $y = x$ and $y = x^2$ in the first octant.

EX 3 Write these integrals as iterated integrals with the order of integration switched.

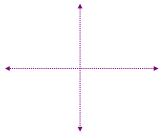
a) $\int_0^2 \int_{y^2}^{2y} f(x,y)\,dx\,dy$



b) $\int_{1/2}^{1} \int_{x^3}^{x} f(x,y)\,dy\,dx$



c) $\int_0^1 \int_{-y}^{y} f(x,y)\,dx\,dy$



EX 4 Evaluate

a) $\int_1^5 \int_0^x \frac{3}{x^2+y^2}\,dy\,dx$

b) $\int_{\pi/6}^{\pi/2} \int_0^{\sin\theta} 6r\cos\theta\,dr\,d\theta$

EX 5 Find the volume of the solid bounded by the parabolic cylinder $x^2 = 4y$ and the planes $z = 0$ and $5y + 9z - 45 = 0$.