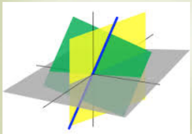
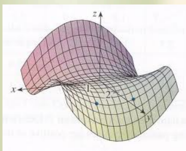
Double Integrals
in
Polar Coordinates



$$f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

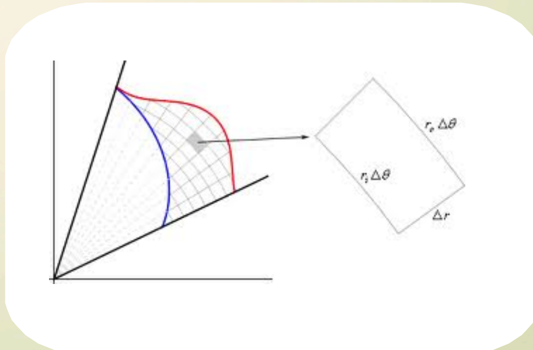
$$f_y = \frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x,y+h) - f(x,y)}{h}$$



$$\int_0^1 \int_0^{2y} xy\,dx\,dy = \int_0^1 \left[\frac{x^2}{2} y\right]_{x=0}^{x=2y} dy$$

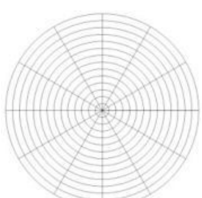
$$= \int_0^1 \frac{(2y)^2}{2} y\,dy = \int_0^1 2y^3\,dy$$

$$= \left[\frac{y^4}{2}\right]_{y=0}^{y=1} = \frac{1}{2}$$

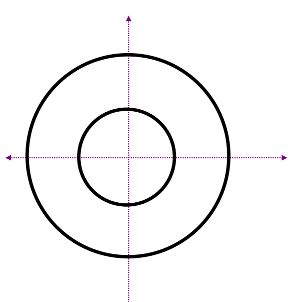


Double Integrals in Polar Coordinates

Rather than finding the volume over a rectangle (for Cartesian Coordinates), we will use a "polar rectangle" for polar coordinates.



$$A_{sector} = \pi r^2 \frac{\Delta\theta}{2\pi} = \frac{1}{2}\,\Delta\theta\, r^2$$



Area of a polar rectangle

Why do we want to integrate polar coordinates?

EX 1 Find the area of the given region S by calculating

$$A = \iint_S dA = \iint_S r\ dr\ d\theta .$$

a) S is the smaller region bounded by $\theta = \pi/6$ and $r = 4\sin\theta$.

EX 1 (cont'd) Find the area of the given region S by calculating

$$\iint_S r\ dr\ d\theta \quad .$$

b) *S is the region outside the circle $r = 2$ and inside the lemniscate $r^2 = 9\cos(2\theta)$.*

EX 2 Evaluate using polar coordinates.

a) $\iint_S y\, dA$ where S is the first quadrant polar rectangle inside $x^2 + y^2 = 4$ and outside $x^2 + y^2 = 1$.

b) $\iint_S (x^2 + y^2)dA$

EX 2 (cont'd) Evaluate using polar coordinates.

c) $\int_0^1 \int_0^{\sqrt{1-y^2}} \sin(x^2 + y^2)\, dx\, dy$