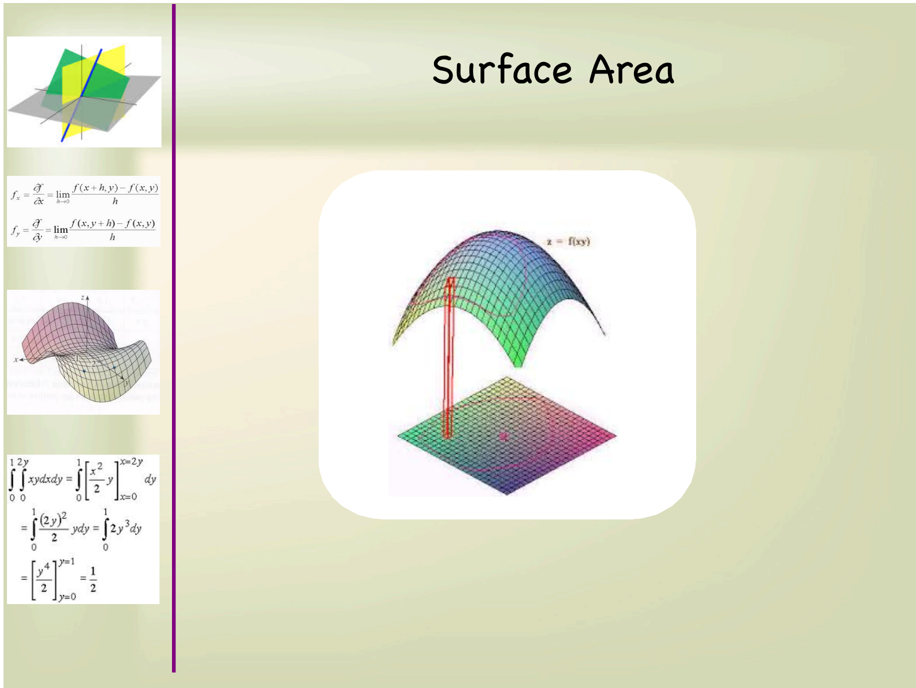
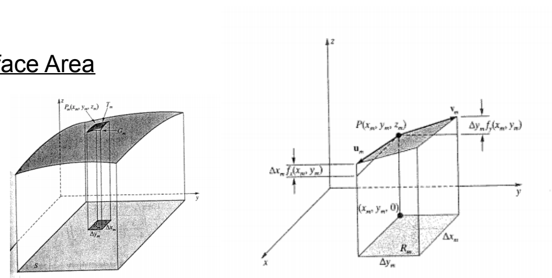


Surface Area



To find the surface area, we are going to add up lots of little areas of parallelograms that are tangent to the surface.

In the limit as Δx and Δy go to zero, the sum becomes an integral which gives the true surface area.

$$\vec{u}_m = \Delta x_m \hat{i} + f_x(x_m, y_m)\Delta x_m \hat{k} = \langle dx_m, 0, f_x(x_m, y_m)dx_m \rangle$$
$$\vec{v}_m = \Delta y_m \hat{j} + f_y(x_m, y_m)\Delta y_m \hat{k} = \langle 0, dy_m, f_y(x_m, y_m)dy_m \rangle$$

We know that the area of the parallelogram is the length of the cross product of its vector sides.

EX 1 Find the surface area of the plane $3x - 2y + 6z = 12$ that is bounded by the planes, $x = 0, y = 0$, and $3x + 2y = 12$.

EX 2 Find the surface area for the part of the sphere, $x^2 + y^2 + z^2 = 9,$ that is inside the circular cylinder, $x^2 + y^2 = 4$.

EX 3 Find the surface area of $z = 4 - x^2 - y^2$
over $S = \{(x,y) | x^2 + y^2 \leq 1\}$.

For a surface area defined parametrically,

$$\vec{r}(u,v) = \langle f(u,v), g(u,v), h(u,v) \rangle .$$

EX 4 Find the surface area of a surface given parametrically by

$$\vec{r}(\theta,\varphi) = \langle 2\sin\varphi\cos\theta, 2\sin\varphi\sin\theta, 2\cos\varphi \rangle .$$

$$R = \{(\theta,\varphi) \mid \theta \in [0,2\pi], \varphi \in [0,\pi]\}$$