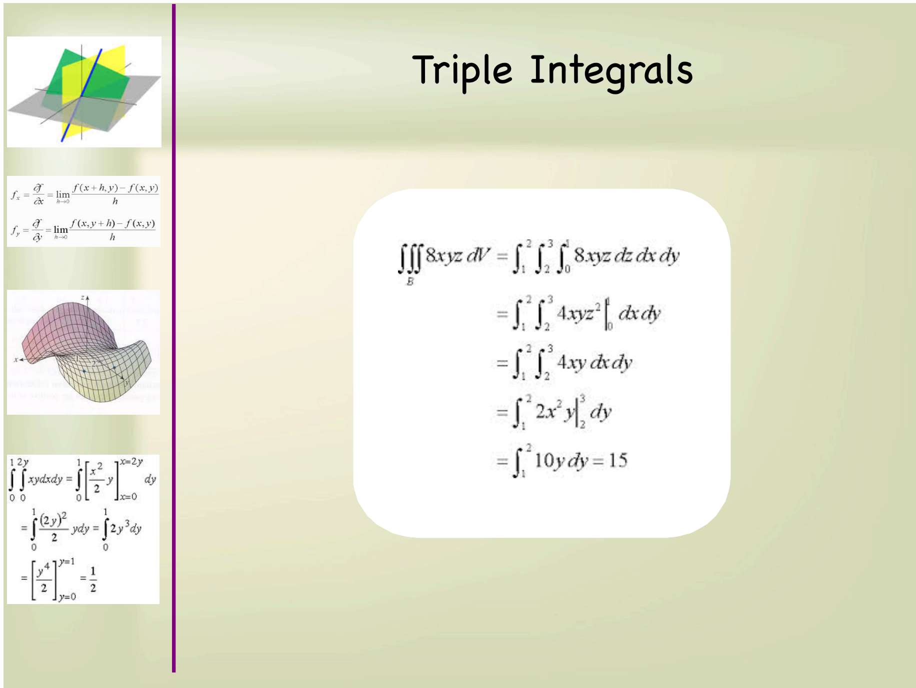


Triple Integrals

$$A = \int_a^b f(x)dx$$ Measures 2-D space (signed area) under a curve above the x-axis.

$$V = \iint_S f(x,y)dA$$ Measures 3-D space (signed volume) under a surface above the xy-plane.

We predict that $\iiint_S f(x,y,z)dV$ measures 4-D space (signed) under a "hyper" surface "above" the xyz-"hyper-plane".

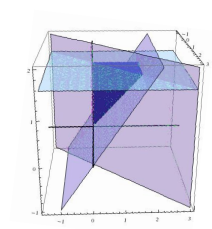
$$\iiint_S f(x,y,z)dV = \int_{a_1}^{a_2}\int_{\varphi_1(x)}^{\varphi_2(x)}\int_{\psi_1(x,y)}^{\psi_2(x,y)} f(x,y,z)dz\,dy\,dx$$

Note: We can't draw anything in 4-D, but we can draw the region S in 3-D (domain space is now 3-D).

EX 1 Write an iterated integral for $\iiint_S (y+z+1)\,dV$

where $S = \{(x,y,z) \mid x \in [0,1],\ y \in [2,5],\ z \in [1,4]\}$.

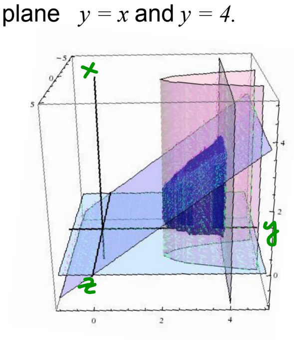
EX 2 Evaluate $\int_0^{\pi/2} \int_0^{z} \int_0^{y} \sin(x+y+z)\,dx\,dy\,dz$.



EX 3 Write an iterated integral for $\iiint_S f(x, y, z)\,dV$

where S is the region in the first octant bounded by the surface $z = 9 - x^2 - y^2$ and the coordinate planes.

EX 4 Find the volume of the solid in the first octant bounded by the hyperbolic cylinder $y^2 - 64z^2 = 4$ and the plane $y = x$ and $y = 4$.



EX 5 Find the volume of the tetrahedron with vertices at (0,0,0), (0,0,3), (0,4,0), and (2,0,0).