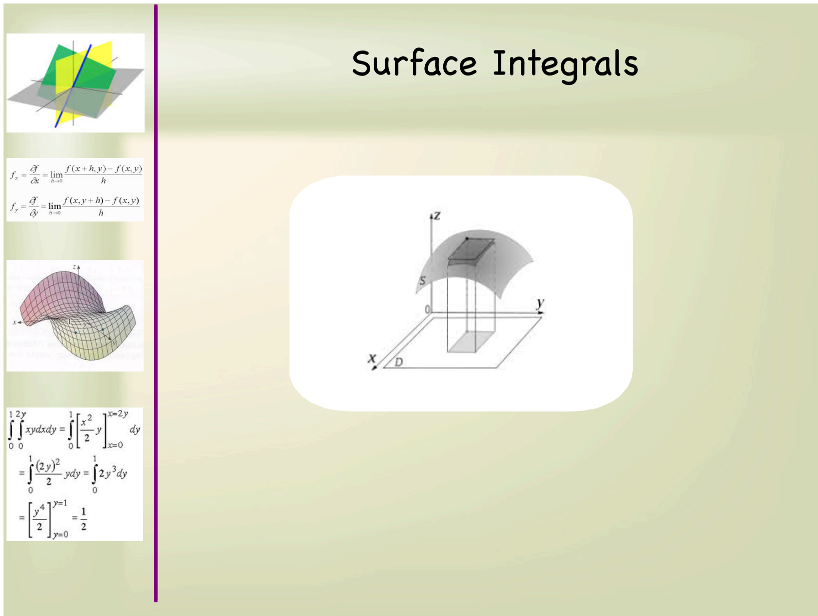


Surface Integrals

Let G be defined as some surface, $z = f(x,y)$.

The surface integral is defined as

$$\int\int_G g(x,y,z)\, dS,$$ where dS is a "little bit of surface area."

To evaluate we need this Theorem:

Let G be a surface given by $z = f(x,y)$ where (x,y) is in R, a bounded, closed region in the xy-plane.

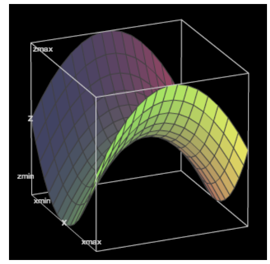
If f has continuous first-order partial derivatives and $g(x,y,z) = g(x,y,f(x,y))$ is continuous on R, then

$$\iint_G g(x,y,z)dS = \iint_R g(x,y,f(x,y))\sqrt{f_x^2 + f_y^2 + 1}\ dy\, dx.$$

EX 1 Evaluate $\int\int_G g(x,y,z)\, dS$ given by $g(x,y,z) = x$, and G is the plane $x + y + 2z = 4$, $x \in [0,1]$, $y \in [0,1]$.

EX 2 Evaluate $\iint_G (2y^2+z)\, dS$ where G is the surface

$z = x^2 - y^2$, with R given by $0 \le x^2 + y^2 \le 1$.



EX 3 Evaluate $\iint_G g(x,y,z)\, dS$ where $g(x,y,z) = z$ and G is the tetrahedron bounded by the coordinate planes and the plane $4x + 8y + 2z = 16$.

Theorem

Let G be a smooth, two-sided surface given by $z = f(x,y)$, where (x,y) is in R and let $\vec{n}$ denote the upward unit normal on G. If f has continuous first-order partial derivatives and $\vec{F} = M\hat{i} + N\hat{j} + P\hat{k}$ is a continuous vector field, then the flux of $\vec{F}$ across G is given by

$$\text{flux } \vec{F} = \iint_G \vec{F}\cdot\vec{n}\, dS = \iint_R [-Mf_x - Nf_y + P]\,dx\,dy\ .$$

EX 4 Evaluate the flux of $\vec{F}$ across G where

$\vec{F}(x,y,z) = (9-x^2)\hat{j}$ and G is the part of the plane

$2x + 3y + 6z = 6$ in the first octant.