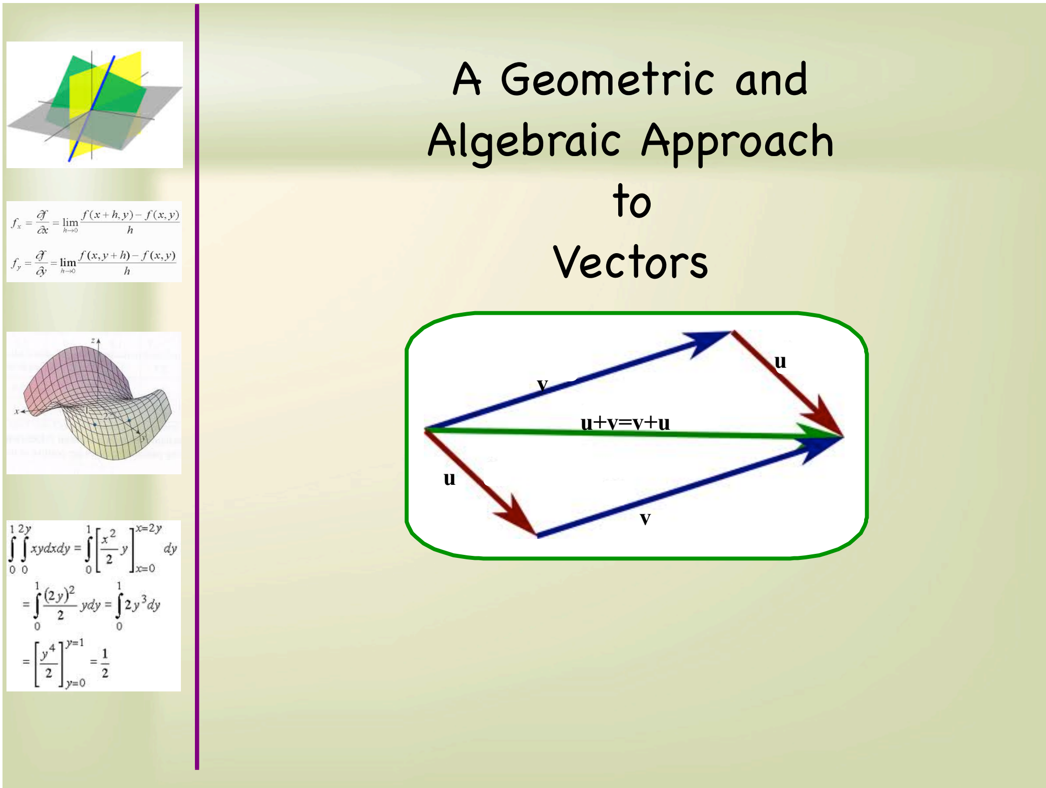


VECTORS (Geometric Approach)

Scalar

Vector

Magnitude

Direction

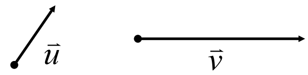
$\vec{u} = \vec{v}$ if they have the same magnitude and direction.

zero vector ⇒ $\vec{0}$ and $\vec{0} + \vec{u} = \vec{u} + \vec{0} = \vec{u}$

$-\vec{u}$ ⇒

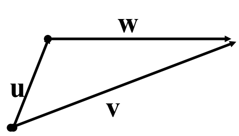
scalar multiple of $\vec{u} \Rightarrow c\vec{u}$, where c is a real number,
means we have a vector in the direction of $\vec{u}$ but scaled in length.

Adding vectors $\Rightarrow$ $\vec{u}+\vec{v}$



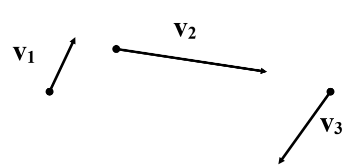
EX 1

Express **w** in terms of **u** and **v**.



EX 2

Draw **w** where $\mathbf{w} = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$



EX 3

Mark pushes on a post in the direction S 30° E with a force of 60 lbs. Dan pushes on the same post in the direction S 60° W with a force of 80 lbs. What are the magnitude and direction of the resulting force?

EX 4

A ship is sailing due south at 20 mph. A man walks west across the deck at 3 mph. What are the magnitude and direction of his velocity relative to the surface of the water?

Vectors (Algebraic Approach)

If we place our vector on a Cartesian Coordinate system with its tail at the origin, then its head will end at some point (u_1, u_2, u_3). We say that $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$

u_1, u_2 and u_3 are called components of $\boldsymbol{u}$.

$\mathbf{u} = \mathbf{v}$ iff $u_1 = v_1,\ u_2 = v_2,\ and\ u_3 = v_3$

$\mathbf{u} + \mathbf{v} = \langle u_1,u_2,u_3 \rangle + \langle v_1,v_2,v_3 \rangle = \langle u_1+v_1,\ u_2+v_2,\ u_3+v_3 \rangle$

$\mathbf{-u} = \langle -u_1, -u_2, -u_3 \rangle \qquad c\mathbf{u} = \langle cu_1, cu_2, cu_3 \rangle$

$\mathbf{0} = 0\mathbf{u} = \langle 0, 0, 0 \rangle$

Theorem A

For all vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ and the real numbers a and b

$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
$\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u}$
$\mathbf{u} + \mathbf{-u} = \mathbf{0}$
$a(b\mathbf{u}) = (ab)\mathbf{u}$
$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$
$(a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}$
$1\mathbf{u} = \mathbf{u}$

$||\mathbf{u}|| = \sqrt{u_1^2+u_2^2+u_3^2}$

$||c\mathbf{u}|| = |c|\ ||\mathbf{u}||$

EX 5

Let $\mathbf{u} = \langle -1, 5, 2 \rangle$, find $||\mathbf{u}||$ and $||-3\mathbf{u}||$.
Also, find a vector, $\hat{\mathbf{u}}$ with the same direction as $\mathbf{u}$ but with magnitude = 1. (This is called a unit vector)