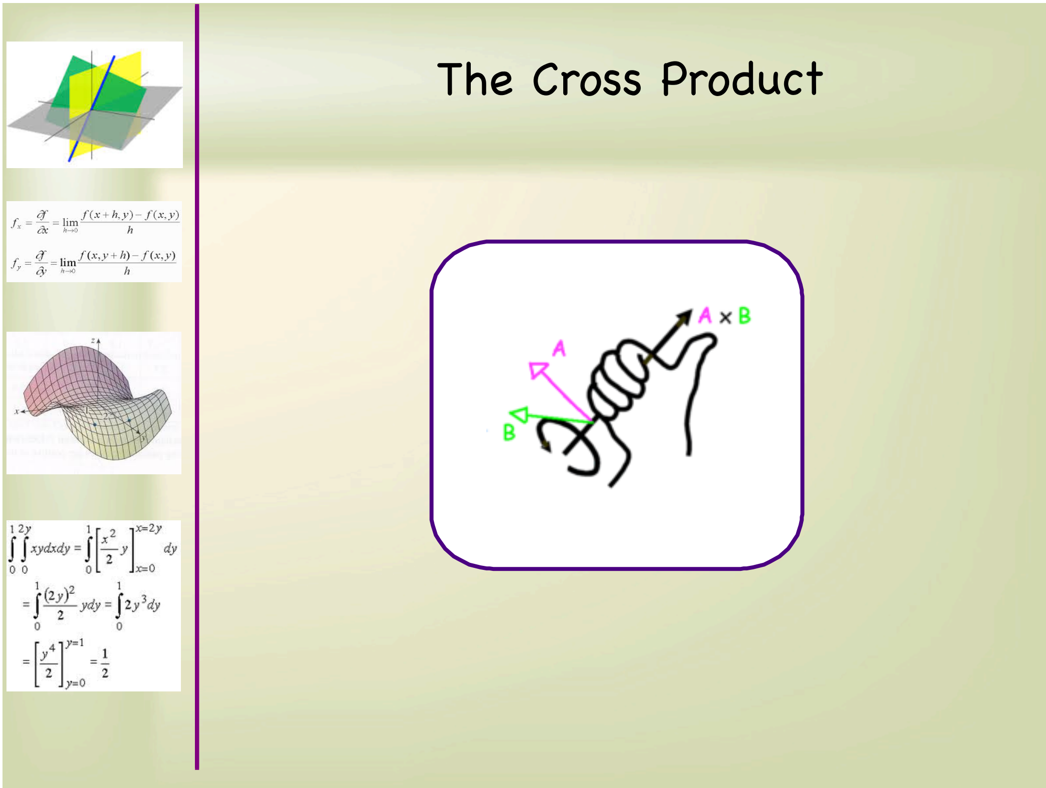


The Cross Product

$$\vec{u}\times\vec{v}=\langle u_2v_3-u_3v_2, u_3v_1-u_1v_3, u_1v_2-u_2v_1\rangle$$

where $\vec{u}=\langle u_1,u_2,u_3\rangle$ and $\vec{v}=\langle v_1,v_2,v_3\rangle$.

EX 1 If $\vec{a}$ = $\langle 3,3,1 \rangle$ and $\vec{b}$ = $\langle -2,-1,0 \rangle$ and $\vec{c}$ = $\langle -2,-3,-1 \rangle$,

find $\vec{a} \times (\vec{b} \times \vec{c})$.

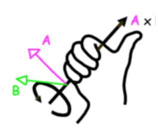
Theorem A

Let $\vec{u}$ and $\vec{v}$ be 3-D vectors and θ is the angle between them.

Then

1) $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0 = \vec{v} \cdot (\vec{u} \times \vec{v})$

2) $\vec{u}, \vec{v}$ and $\vec{u} \times \vec{v}$ form a right-handed triple.



3) $\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \|\vec{v}\| \sin \theta$

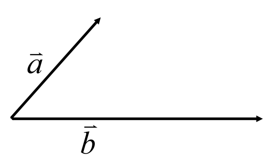
Theorem B

Two 3-D vectors, $\vec{u}$ and $\vec{v}$ are parallel iff $\vec{u} \times \vec{v} = 0$.

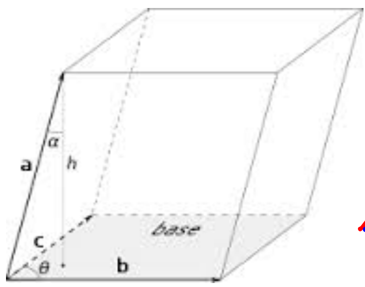
EX 2 Find the plane through these points.

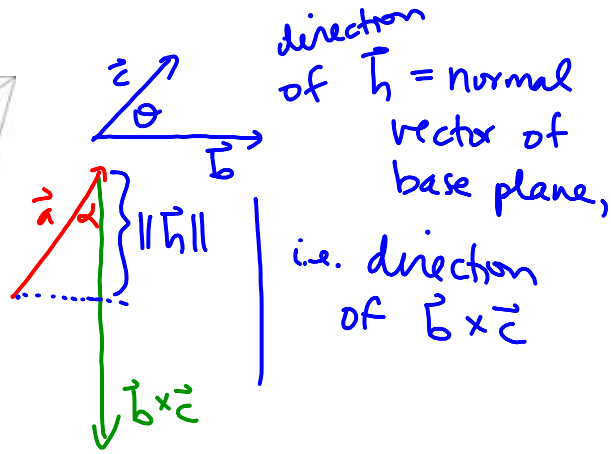
$P_1(-1,3,0)$, $P_2(5,1,2)$ and $P_3(4,-3,-1)$

EX 3 Find the area of a parallelogram with vectors $\vec{a}$ and $\vec{b}$ as adjacent sides.



EX 4 Find the volume of a parallelogram prism(box) determined by the sides $\vec{a}$, $\vec{b}$ and $\vec{c}$.





Theorem C Properties of Cross Product

$\vec{u}$, $\vec{v}$ and $\vec{w}$ are 3-D vectors and $k \in \Re$:

1) $\vec{u}\times\vec{v} = -\vec{v}\times\vec{u}$

2) $u\times(v+w) = (\vec{u}\times\vec{v}) + (\vec{u}\times\vec{w})$

3) $k(\vec{u}\times\vec{v}) = (k\vec{u})\times\vec{v} = \vec{u}\times(k\vec{v})$

4) $\vec{u}\times\vec{0} = \vec{0}\times\vec{u} = \vec{0}$ and $\vec{u}\times\vec{u} = \vec{0}$

5) $(\vec{u}\times\vec{v})\cdot\vec{w} = \vec{u}\cdot(\vec{v}\times\vec{w})$

6) $\vec{u}\times(\vec{v}\times\vec{w}) = (\vec{u}\cdot\vec{w})\vec{v} - (\vec{u}\cdot\vec{v})\vec{w}$

$$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{j}\times\hat{k} = \hat{i} \qquad \hat{k}\times\hat{i} = \hat{j}$$

EX 5 State whether each of the following expressions make sense or not. If it makes sense, tell if the result is a scalar or a vector.

a) $\vec{u} \cdot (\vec{v} \times \vec{w})$

b) $\vec{u} + (\vec{v} \times \vec{w})$

c) $(\vec{a} \cdot \vec{b}) \times \vec{c}$

d) $(\vec{a} \times \vec{b} + k)$

e) $(\vec{a} + \vec{b}) \times (\vec{c} + \vec{d})$

f) $(\vec{a} \cdot \vec{b} + k)$

EX 6 Find the equation of a plane through $(5,-1,2)$ that is perpendicular to the line of intersection of the planes $4x-3y+2z=1$ and $2x-y+z=11$.