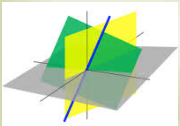
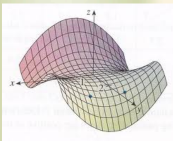


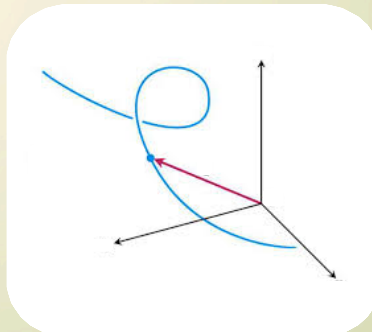
$$f_x = \frac{\partial f}{\partial x} = \lim_{h\to 0}\frac{f(x+h,y)-f(x,y)}{h}$$

$$f_y = \frac{\partial f}{\partial y} = \lim_{h\to 0}\frac{f(x,y+h)-f(x,y)}{h}$$



$$\int_0^1\int_0^{2y} xy\,dx\,dy = \int_0^1\left[\frac{x^2}{2}y\right]_{x=0}^{x=2y}dy = \int_0^1\frac{(2y)^2}{2}y\,dy = \int_0^1 2y^3\,dy = \left[\frac{y^4}{2}\right]_{y=0}^{y=1} = \frac{1}{2}$$

Vector-Valued Functions and Curvilinear Motion



A vector-valued function associates a vector output, $\vec{F}(t)$,

to a scalar input.

i.e $\vec{F}(t) = f(t)\hat{i} + g(t)\hat{j} = \langle f(t), g(t)\rangle$ (in 2D)

where f and g are real-valued functions of t

or (in 3D).

Definition $\lim_{t\to c}\vec{F}(t) = \vec{L}$ means that for every $\varepsilon > 0$ there is a corresponding $\delta > 0$ such that $\|\vec{F}(t) - \vec{L}\| < \varepsilon$, provided $0 < |t-c| < \delta$, i.e.

$$0 < |t-c| < \delta \Rightarrow \|\vec{F}(t) - \vec{L}\| < \varepsilon$$

.

Theorem A Let $\vec{F}(t) = f(t)\hat{i} + g(t)\hat{j}$. Then $\vec{F}$ has a limit at c iff f and g have limits at c *and*

$$\lim_{t\to c} \vec{F}(t) = \left[\lim_{t\to c} f(t)\right]\hat{i} + \left[\lim_{t\to c} g(t)\right]\hat{j}$$

Continuity ⇒ $\vec{F}(t)$ is continuous at $t = c$ if $\lim_{t\to c} \vec{F}(t) = \vec{F}(c)$.

Derivative ⇒ $\vec{F}'(t) = \lim_{h\to 0} \dfrac{\vec{F}(t+h) - \vec{F}(t)}{h}$

Differentiation Formulas

$\vec{F}(t)$ & $\vec{G}(t)$ are differentiable

$c \in \Re$

$h(t)$ is differentiable

1) $D_t[\vec{F}(t) + \vec{G}(t)] = \vec{F}'(t) + \vec{G}'(t)$
2) $D_t\left[c\vec{F}(t)\right] = c\vec{F}'(t)$
3) $D_t[h(t) \cdot \vec{F}(t)] = h(t) \cdot \vec{F}'(t) + h'(t) \cdot \vec{F}(t)$
4) $D_t[\vec{F}(t) \cdot \vec{G}(t)] = \vec{F}(t) \cdot \vec{G}'(t) + \vec{F}'(t) \cdot \vec{G}(t)$
5) $D_t[\vec{F}(h(t))] = \vec{F}'(h(t)) \cdot h'(t)$

Integration Formula

$$\int \vec{F}(t)dt = \left[\int f(t)dt\right]\hat{i} + \left[\int g(t)dt\right]\hat{j}$$

Ex 1 Find $\lim_{t \to \infty} \left[\frac{t \sin t}{t^2} \hat{i} - \frac{7t^3}{t^3 - 3t} \hat{j} \right]$.

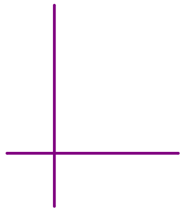
EX 2 Find $\vec{F}'(x)$ and $\vec{F}''(x)$ for $\vec{F}(x) = (e^x + e^{-x^2})\hat{i} + \cos(2x)\hat{j}$.

EX 3 $\vec{f}(y) = (\tan^2 y)\hat{i} + \sin^2(\tan^2 y)\hat{j} + 3y\,\hat{k}$

Find $\vec{f}'(y)$.

$\vec{r}(t)$ is position vector at any time t along a curve given by

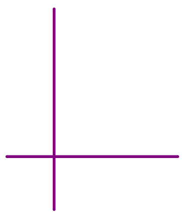
$x = x(t)$ and $y = y(t)$.



$$\vec{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j}$$

$$\vec{v}(t) = \vec{r}\,'(t) = x'(t)\hat{i} + y'(t)\hat{j}$$

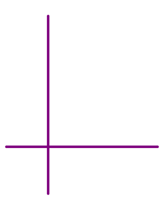
$$\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t) = x''(t)\hat{i} + y''(t)\hat{j}$$



EX 4 Given $\vec{r}(t) = (4\sin t)\,\hat{i} + (8\cos t)\,\hat{j}$,

a) Find $\vec{v}(t)$ and $\vec{a}(t)$.

b) Find the speed when $t = \pi/4$.



c) Sketch a portion of the graph of $\vec{r}(t)$ containing the position P of the particle at $t = \pi/4$. Draw $\vec{v}$ and $\vec{a}$ at P as well.

EX 5 Suppose that an object moves around a circle with center at $(0,0)$ and radius r at a constant angular speed of ω radians/sec. If its initial position is $(0,r)$, find its acceleration.