

MATH CIRCLE CONTEST I
October 21, 2009

1.

Let $n = 10212009$. Prove that there are two distinct whole numbers of the form

$$111 \cdots 111000 \cdots 000$$

whose difference is divisible by n . (For instance, if $n = 2$ instead, we could take the numbers to be 1 and 11 since $11 - 1$ is divisible by 2.)

2.

Fifteen chairs are evenly placed around a circular table on which are name cards for fifteen guests. The guests fail to notice these cards until after they have been seated. It turns out that no one is seated in front of the card with their name on it. Prove that the table can be rotated so that at least two of the guests are simultaneously seated in front of their own name card.

3.

Prove that, given 5 points on the surface of a sphere, there exists a closed hemisphere which contains at least 4 of them.

4.

Consider the Fibonacci sequence

$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \dots$

Prove there are infinitely many terms in the sequence which are divisible by 7. Is there anything special about 7?