MATH CIRCLE CONTEST III

February 25, 2004

EISENSTEIN'S REVENGE

(a) Prove or disprove: $x^5+34x^4+287x^3+867x^2+85$ is irreducible over the rational numbers.

(b) Prove or disprove: $x^4-4x^3-15x^2-5x+5$ is irreducible over the rational numbers.

Consider the following game. The numbers $1, \ldots, 16$ are initially arranged in order. An allowable move consists of swapping the numbers in position i and $i+3$ and simultaneously swapping those in the $i+1$ and $i+2$ position; here $1 \leq i \leq 13$. For instance, if $i = 3$, the move takes the initial configuration

$$1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10\ \ 11\ \ 12\ \ 13\ \ 14\ \ 15\ \ 16$$

to

$$1\ \ 2\ \ 6\ \ 5\ \ 4\ \ 3\ \ 7\ \ 8\ \ 9\ \ 10\ \ 11\ \ 12\ \ 13\ \ 14\ \ 15\ \ 16$$

Prove or disprove: every sequence of the numbers $1, \ldots, 16$ can be obtained from the initial configuration through a series of allowable moves.

SWAPPING GAMES, PART 2

The situation similar to the first problem, but there are now only the numbers $1, \ldots, 8$ and the moves are different. An allowable move now consists of swapping the number in the i position with the one in the $i+1$ position; here $1 \leq i \leq 7$. For instance if $i = 3$, the move takes

$$1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8$$

to

$$1\ \ 2\ \ 4\ \ 3\ \ 5\ \ 6\ \ 7\ \ 8.$$

Michelle the mathematician finds that she can move from the initial configuration

$$1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8$$

to

$$8\ \ 7\ \ 6\ \ 1\ \ 3\ \ 4\ \ 5\ \ 2$$

in 25 moves. Prove or disprove: there is no shorter sequence of allowable moves taking the initial configuration to the one given above.