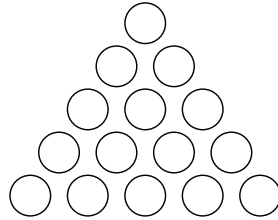


MATH CIRCLE FINALE
April 28, 2004

1. My watch runs 1 second fast per hour. Eric's runs 1.5 seconds slow per hour. Right now they both read the same time (which is the correct time).
 - (a) When will our watches read the same time again?
 - (b) When will our watches read the same *correct* time again?

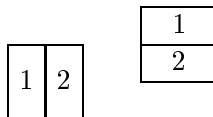
2. A new internet start-up employs 36 employees. The office is located on the first floor of a square building which is 60 feet by 60 feet. Each employee has a 10-by-10 cubicle, and each cubicle has doors to each adjacent cubicle. The cubicles in the northeast and southwest corners of the building also have exits out of the building. The mailman wants to start at one entrance, visit each cubicle exactly once, and leave out the other exit. Can he do it? (Justify your answer.)

3. Suppose 15 pennies are arranged in a triangle, as in the following diagram:



Show that inside this triangle, there is guaranteed to be an equilateral triangle in which all the vertex pennies are facing the same way (either all heads up or all tails up).

4. Suppose you are given $2n$ 2-by-1 dominos labeled $1, 2, \dots, 2n$. How many ways can you arrange them in a 2-by- $2n$ rectangle so that the numbers on the dominos increase along rows and columns of the rectangle. For instance, when $n = 2$ we have two possibilities



and when $n = 4$ we have six:

