1. AN ESTHETIC TOUCH

A Madison Avenue marketing guru has discovered that the most eye-catching billboard is one
so that the square of the area plus the area itself is equal to the length to the fourth power
minus the width to the fourth power plus one-eighth the square of the perimeter, minus
one-half the sum of the squares of the length and the width. If the width of a particular
billboard is to be one meter, what should its length be? (You need not reduce your answer
to an actual numerical value; leaving it in a form analogous to $\frac{2+\sqrt{2}}{2}$ is acceptable.)

2. TEMPLE SQUARE TO FOOTHILL VILLAGE (ROUGHLY)

Suppose you inhabit a strange city whose streets are laid out according to a strict grid pattern. You begin at the corner of 0th Street South and 0th Street East and by walking only South and East you seek to arrive at 17th South and 21st East. The catch is that you have to pass through the intersection of 9th South and 9th East as well as the intersection of 15th South and 15th East. How many different paths are there to your destination? (You need not reduce your answer to an actual numerical value; leaving it in a form analogous to $(100!)/3^7 + 20!/2^8$ is acceptable.)

3. RETURN OF FIBONACCI

Let $f_1, f_2, f_3, \ldots$ denote the familiar Fibonacci sequence defined by taking $f_1 = f_2 = 1$ and for $n \geq 3$, $f_n = f_{n-1} + f_{n-2}$. Prove that

$$f_1^2 + f_2^2 + \cdots + f_n^2 = f_{n+1} f_n.$$

4. PIGEONS ANYONE?

Prove that there exists a multiple of 1997 whose decimal expansion contains only digits 1 and 0.