

MATH CIRCLE CONTEST II
December 6, 2006

1. DINNER IS SERVED

Eight guests arrive at a restaurant and are seated at a circular table. Before leaving, each guest jots down the names of those seated next to him. Over dessert, a mathematically inclined guest poses the following question: how many ways could the guests have been seated that evening so that each left with the identical list of names?

2. AT THE RACES

How many distinct ways can five horses cross the finish line of a race? (Ties are allowed!)

3. DRAWING BOUNDARIES

Suppose six lines are drawn in the plane. What is the maximal number of regions in the plane that are cut out by the lines?

4. CHORD PROGRESSIONS

Eight distinct points are chosen on a circle. In how many ways can pairs of these eight points be joined by non-intersecting line segments?

5. EASY AS ONE, TWO, THREE

Can one arrange the numbers $1, 2, \dots, 9$ along a circle so that the sum of two neighbors are never divisible by 3, 5, or 7?