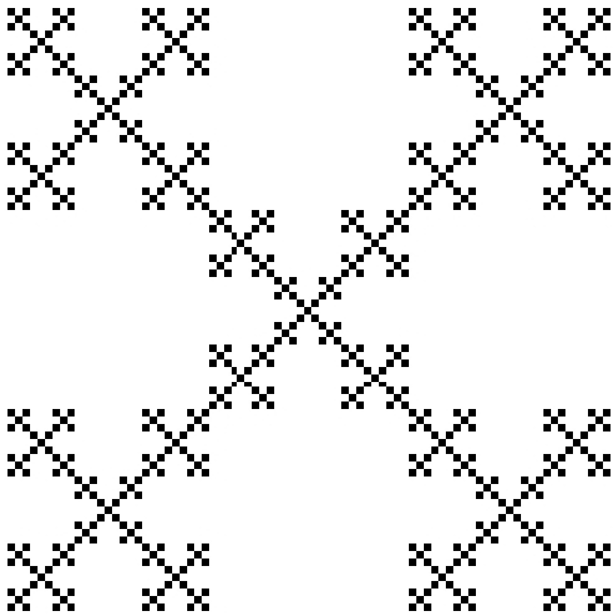
Utah Math Circle. Contest 2. Fall 2008.

Name: ______________________ December 10, 2008

Problem 1. Compute the box-counting dimension for the following fractal:



Recall that, by definition, the box-counting dimension is

$$\frac{\log_3 N(n, K)}{n}$$

as n goes to infinity, where $N(n, K)$ is the number of 3^{2n} sub-boxes intersecting the fractal K. (It is better for this fractal to use $\log_3$ rather than $\log_2$.)

Problem 2. Assume $2n$ points in the plane are given, no three which are collinear. Show that there exists a halving line, that is, a line going through two points such that $n-1$ points are on one side and $n-1$ are on the other side of the line.

Extra credit: Prove that in fact there are at least n different halving lines.

Problem 3. What is the minimum number of elements of the set $\{1, 2, 3, \ldots, 19\}$ we should select randomly, to guarantee that there are at least two which sum up to 20?

Problem 4. Recall Pick's Theorem which says that the area A of any lattice simple polygon is given in terms of the formula

$$A = I + \frac{B}{2} - 1,$$

where I is the number of lattice points in the interior of the polygon, and B is the number of lattice points on the boundary of the polygon.

Prove that any lattice $n \times n$ square cannot cover more than $(n+1)^2$ lattice points.