

HOW ABOUT GETTING A LARGER BOAT?

A Math Circle session by Renzo

Abramo Pastore, a farmer from the quaint little village of La Pasta, is walking to the larger city of Minestrone to have his pets, Ugo the lion and Aldo the llama, checked by the vet. As a gift for the vet he is also carrying one of his world famous homegrown heads of lettuce.

As he approaches the river he notices, alas, that the little wooden bridge has collapsed. Fortunately, a small boat is resting on Abramo's side of the river.

"Maybe not all is lost!" exclaims Abramo, and he jumps on the boat, soon to realize that there is no way to fit Ugo, Aldo, the lettuce (remember, it's a HUGE lettuce) and himself in the boat at the same time. As a matter of fact, not even two of the three items will fit with him. He must carry things over one at a time.

And here is the problem. Aldo and Ugo cannot be left alone...they are not exactly best friends, as Ugo tends to see Aldo as "llama chops" rather than just llama. On the other hand, Aldo would just LOVE to be left alone with the lettuce, but that would mean no lettuce for the vet and a llama to be cured for indigestion.

Apparently there seems to be nothing to do but turn around and go home; but Abramo never in his life has let his strong Italian temper get discouraged and has given up in front of hardships.

"There must be a way out! I just feel it " he thinks. He takes out his notepad, scribbles a few things, draws a few diagrams and then jumps up satisfied.

"Mamma mia! That was quite easy after all!!".

What did Abramo think of?

So Abramo gets to the vet's house in Minestrone. After Aldo and Ugo get checked up he's ready to head back home when he notices a furry and rather monstrous animal in a cage.

"What is that?" asks Abramo.

“That’s a leviathan. I am trying to get rid of him. It’s an extremely nice beast; but there is a catch. It attacks lions, unless there’s also lettuce around. Do you want it? I’ll give you your lettuce back if you want it. You’ll do me a great favor if you take it”.

“I’d love to take it, except I want to make sure I can cross the river” and Abramo explains the situation to the vet.

“There seems to be no way out then...” sadly says the vet.

“Maybe, maybe...”ponders Abramo, taking his notepad out again and drawing some diagrams that the vet could not comprehend...

What do you think?

TWO METHODS FOR SOLVING ABRAMO’S PROBLEMS.

Let us analyze in detail the first problem, where only Lion, Llama and Lettuce need to be transported. It will be up to you to generalize to the more complicated case when also the Leviathan appears.

We will assign to each of the creatures either the number 0 or 1. 0 will correspond to being on the first side of the river, 1 to being on the opposite side.

FOR GEOMETRY LOVERS:

Let’s pick three cartesian axes and label them *Lion*, *Llama*, *Lettuce*. A point in *LLL* space is given by three coordinates (a, b, c) . In particular we are interested in those points where a, b and c are either 0 or 1.

We have eight such points, each of them representing a different configuration of the creatures. For example the point $(0, 1, 0)$ corresponds to the situation: the lion and the lettuce are on this side of the river, the llama on the other.

Then my goal is to move from the point $(0, 0, 0)$ to the point $(1, 1, 1)$. But I can’t just move arbitrarily. For example I can’t just connect the two points with a segment and go along it. I have to obey to the following prescriptions:

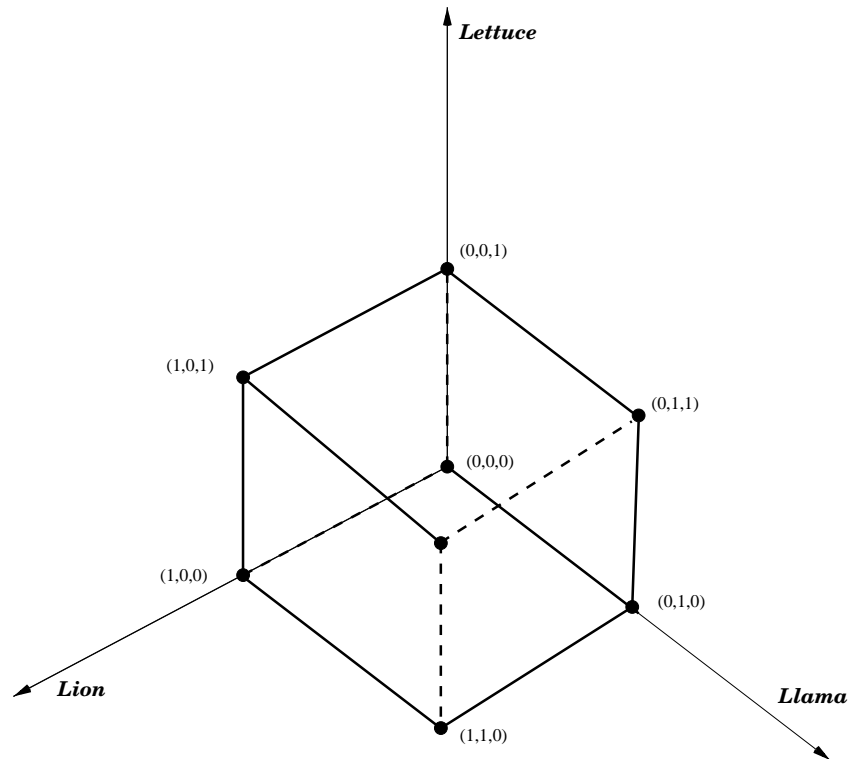
1. I can only move one animal at a time. This means that, in *LLL* space,

I can only change one coordinate at a time, i.e. I can only move along the sides of the cube generated by my eight points.

2. The lion eats the llama and the llama eats the lettuce. This means that during any trip I can't have the lion and the llama coordinates or the llama and the lettuce coordinates remain the same. This brings me to declare some of the sides of the cube to be "off-limits".

NOTICE: condition two is a condition on sides and not on vertices. I can be at a vertex where the lion and the llama coordinates are the same, because at a vertex Abramo is there to watch over them. The coordinates can't remain the same during a trip, i.e. along a side.

We have then reduced the problem to the following geometrical problem: can I move along the sides of the cube from point $(0, 0, 0)$ to point $(1, 1, 1)$ avoiding the "evil" sides (that in the following figured are dashed)?



It's now clear that there are two possible distinct solutions, and that the minimum number of moves to be made is 5.

FOR GRAPH THEORY LOVERS:

There are eight possible configurations of the three creatures. To each of them let's assign a dot and use the coordinate convention established before to label the configuration efficiently.

Let us arrange the dots in columns according to the sum of the coordinates. Now, again, my goal is to move from the leftmost to the rightmost point. But I do have the following restrictions:

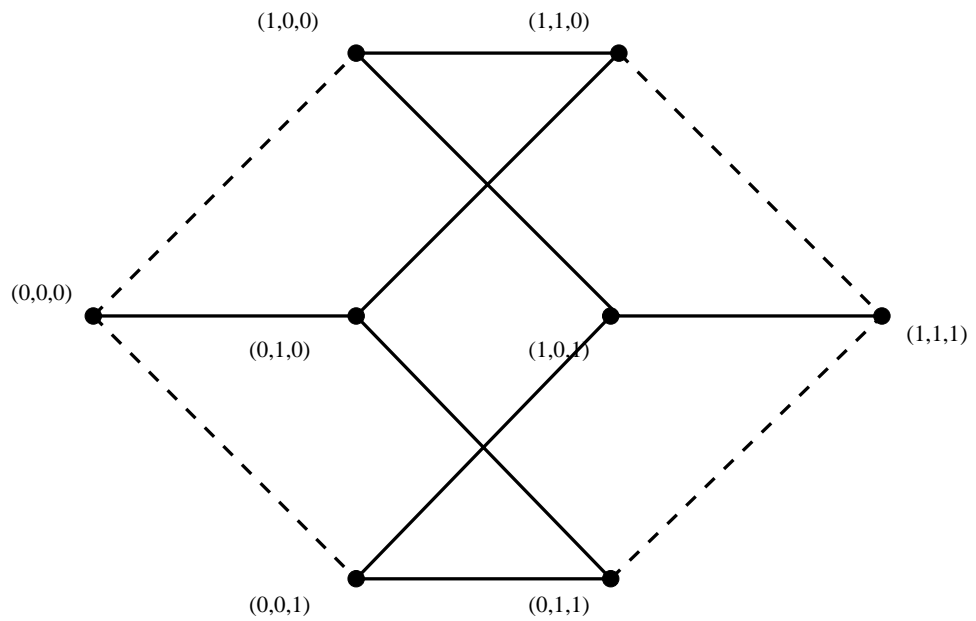
1. I can only change one coordinate at a time. In particular, I can only move from one column to one of the neighbouring ones. And notice, even so, not all moves are allowed. For example I cannot move from $(0, 1, 0)$ to $(1, 0, 1)$.
2. At any transition I can't maintain the lion and the llama coordinates to be equal, nor the llama and the lettuce coordinates.

Now the procedure is simple. Start from $(0, 0, 0)$ and try to move along the graph, following the above prescriptions, and to get to the point $(1, 1, 1)$.

Furthermore, if you complete the graph, i.e. if you check all possible admissible moves, you can tell from the graph the number of solutions and what is the "most efficient" solution.

In the following picture I drew all the lines that satisfy condition (1). The solid lines satisfy also condition (2), whereas the dashed ones don't.

(Lion, Llama, Lettuce)



Now it's up to you to figure out the solutions to the problem when the leviathan appears in the picture!!

GOOD LUCK!

Inspiration and ideas for this handout come mostly from Ian Stewart's book "Another fine math you've got me into", Chapter 1.