MATH CIRCLE CONTEST III
February 15, 2006

1. Temple Square to East High

Suppose you're at Temple Square (which is zero South and zero East) and you want to travel to the East High School at 13th East and 9th South. The hitch is that you can only travel East and South along numbered streets. (Assume that all numbered streets exist and that they are all two-way streets.) How many different routes can you take to East High?

2. square drill bits

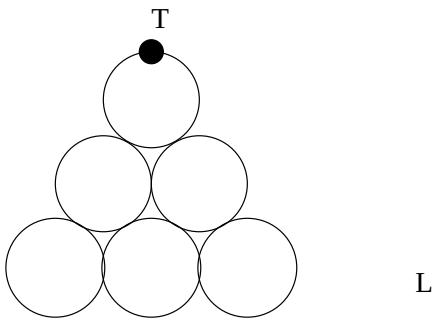
Consider a cube of side length three. Drill square holes of side length one in each face. What is the surface area of the resulting object?

3. powers of 2

In the prime factorization of $50! = 50 \cdot 49 \cdot 48 \cdots 2 \cdot 1$ what is the power of 2 that appears? (For instance, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 = 2^3 \cdot 3 \cdot 5$. So 3 is the power of 2 that appears in the prime factorization of 5!.)

4. SIXPENCE NONE THE RICHER

Consider the following arrangement of six pennies.



If a penny has radius of 1 centimeter, how high is the point T above the line L?

5. POWER ALIGNMENT

(a) Find the smallest positive whole number that is twice a fifth power and five times a perfect square.
(b) Find the smallest positive whole number that is twice a perfect square and three times a perfect cube.