

MATH CIRCLE CONTEST IV  
March 8, 2006

1. BREAKING THE BANK I

A bank customer asks for one dollar in pennies, dimes, and nickels. The teller (who has an infinite supply of coins) asks how many of each coin he would like. The customer responds that any combination would do. How many different ways can the teller fulfill the customer's request?

## 2. MAILBOXES ET CETERA

A new internet startup occupies an office space that is perfectly square and whose walls are aligned with the directions on a compass. The square office space is divided into 100 identical square cubicals. Each cubical shares a door with each adjacent cubical. In addition there is an entrance to the cubical in the Northwest corner of the building and an exit on the Southeast corner. Mike (the mathematical mailman) wants to enter the building through the Northwest entrance, visit every cubical exactly once, and depart out the Southeast exit. Can he do it? Either devise a path for him to follow or prove that no such path exists.

### 3. BREAKING THE BANK II

The situation is the same as Problem 1, but this time assume that the customer asks for ten dollars in pennies, dimes, and nickels. In how many ways can the teller fulfill the customer's request?

#### 4. TROMINOS AND CHESSBOARDS

Consider a standard eight-by-eight chessboard. Suppose you are given a bag of “trominos”. These are just like dominos except that they look like



where the size of each box is the same size as a box on the chessboard. Now you are asked to tile the chessboard using the bag of trominos in such a way that only the upper-left corner is left untiled. Can you do it? Either find a tiling or prove no such tiling exists.