HW6 1/2

\# 7.2.7

$$F(\mu) = \int_{-\infty}^{+\infty} \frac{\sin \pi x}{\pi x} e^{i\mu x}\, dx = \int_{-\infty}^{+\infty} \frac{e^{i(\mu+\pi)x} - e^{+i(\mu-\pi)x}}{2\pi i x}\, dx$$

According to the table on page 335,

$$H(x) \xrightarrow{\mathcal{F}} \frac{1}{-i\mu} \quad \text{i.e. } \frac{1}{2\pi} \int \frac{1}{-i\mu} e^{-i\mu x}\, d\mu = H(x)$$

Just change notations $(\mu, x) \to (x, \mu)$:

$$\frac{1}{2\pi} \int \frac{1}{-ix} e^{-i\mu x}\, dx = H(\mu) \xRightarrow[\text{conj.}]{\text{complex}} \frac{1}{2\pi} \int \frac{1}{ix} e^{i\mu x}\, dx = H(\mu)$$

So,

$$\int_{-\infty}^{+\infty} \frac{e^{i(\mu+\pi)x}}{2\pi i x}\, dx = H(\mu+\pi) \qquad \int_{-\infty}^{+\infty} \frac{e^{i(\mu-\pi)x}}{2\pi i x}\, dx = H(\mu-\pi)$$

$$F(\mu) = H(\mu+\pi) - H(\mu-\pi) = \begin{cases} 1, & |\mu| < \pi \\ 0, & |\mu| > \pi \end{cases}$$

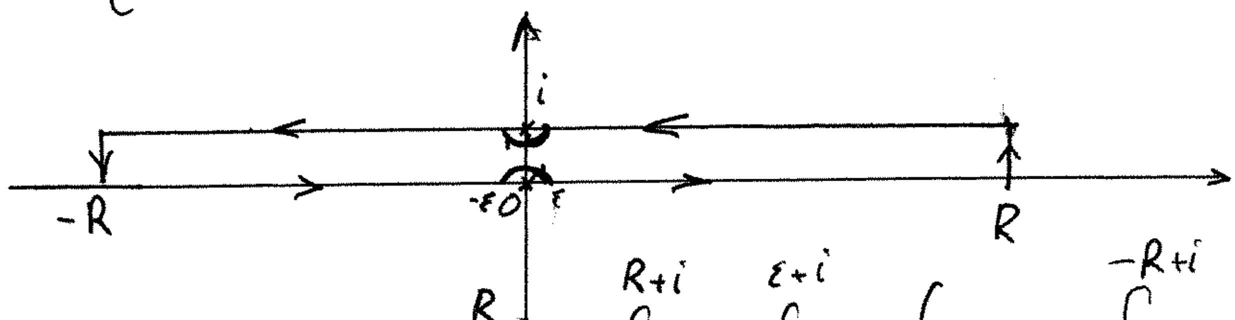
HW 6 2/2

6.4 #13

$$I = \int_0^{\infty} \frac{\sin \alpha x}{\sinh \pi x}\, dx = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{\sin\alpha x}{\sinh \pi x}\, dx = \frac{1}{2}\,\mathrm{Im}\int_{-\infty}^{+\infty} \frac{e^{i\alpha x}}{\sinh \pi x}\, dx$$

More precisely, the principal value is included in the sign of this integral

$$I' = \oint_C \frac{e^{i\alpha z}}{\sinh \pi z}\, dz =$$



$$0 = \int_{-R}^{-\varepsilon} + \int_{|z|=\varepsilon} + \int_{\varepsilon}^{R} + \int_{R}^{R+i} + \int_{R+i}^{\varepsilon+i} + \int_{|z-i|=\varepsilon} + \int_{-\varepsilon+i}^{-R+i} + \int_{R+i}^{R}$$

$\int_{|z|=\varepsilon}$: arg z goes from π to 0; $\xrightarrow{\varepsilon\to 0} -i$

$\int_{R}^{R+i}$: $\xrightarrow{R\to\infty} 0$

$\int_{|z-i|=\varepsilon}$: arg$(z-i)$ goes from 0 to $-\pi$; $\xrightarrow{\varepsilon\to 0} +ie^{-\alpha}$

$\int_{R+i}^{R}$: $\xrightarrow{R\to\infty} 0$

$$\int_{-\infty}^{+\infty} \frac{e^{i\alpha x}}{\sinh \pi x}\, dx - \int_{-\infty}^{+\infty} \frac{e^{i\alpha(x+i)}}{\sinh[\pi(x+i)]}\, dx = i\left(1-e^{-\alpha}\right)$$

$$\int_{-\infty}^{+\infty} \frac{e^{i\alpha x}}{\sinh \pi x}\, dx \times \left[1 + e^{-\alpha}\right] = i\left(1-e^{-\alpha}\right)$$

$$\int_{-\infty}^{+\infty} \frac{e^{i\alpha x}\,dx}{\sinh \pi x} = i\,\frac{1-e^{-\alpha}}{1+e^{-\alpha}} = i\tanh\frac{\alpha}{2} \Rightarrow I = \frac{1}{2}\tanh\frac{\alpha}{2}$$