1210-90 Exam 1
Fall 2012

Name KEY

Instructions. Show all work and include appropriate explanations when necessary. Please try to do all work in the space provided. Please circle your final answer.

1. (20pts) Compute the following limits. Be sure to show your work. **Note:** Answers can be values, $+\infty$, $-\infty$, or DNE (does not exist)

4 pts each partial credit possible

(a) $\lim_{x\to 3}(x^2 - x + 5)$ $= \lim_{x\to 3} x^2 - \lim_{x\to 3} x + \lim_{x\to 3} 5$

$= 9 - 3 + 5 = 11$

(b) $\lim_{x\to 3}\frac{x^2-9}{x-3}$ $= \lim_{x\to 3}\frac{(x+3)(x-3)}{x-3} = \lim_{x\to 3} x+3 = 6$

(c) $\lim_{x\to 0} x\cos(4x)$ $= \left(\lim_{x\to 0} x\right)\left(\lim_{x\to 0}\cos(4x)\right) = 0 \cdot 1 = 0$

(d) $\lim_{x\to 0^+}\frac{x^2-1}{x}$

when $x>0$ and x is near zero, x^2-1 is near -1 and x is near zero. Furthermore $-1/x < 0$ and so

$$\lim_{x\to 0^+}\frac{x^2-1}{x} = -\infty$$

(e) $\lim_{x\to 0}\frac{x^2-1}{x}$

Since $\lim_{x\to 0^-}\frac{x^2-1}{x} = +\infty$ by similar reasoning,

$$\lim_{x\to 0}\frac{x^2-1}{x} \text{ DNE}$$

2. (20 pts) Compute the following derivatives.

(a) $D_x(x^3 + 2x)$

$= 3x^2 + 2$

4 pts each
partial credit
possible

(b) $D_x(\frac{x}{x-1})$

$= \frac{(x-1)\cdot 1 - x(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$

(c) $D_x(x^2 \cos(x))$

$= 2x\cos x - x^2 \sin x$

(d) $D_x((x^2 + x)^9)$

$= 9(x^2+x)^8(2x+1)$

(e) $D_x(\sin^4(x^3))$ **Note:** $\sin^4(x^3)$ is the same as $(\sin(x^3))^4$.

$= 4\sin^3(x^3)\cos(x^3)\cdot 3x^2$

$= 12x^2\sin^3(x^3)\cos(x^3)$

3. (10pts) Compute the derivative of $f(x) = x^2 + x$ by using **the definition of the derivative**; that is, compute the limit

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + (x+h) - x^2 - x}{h} \quad 3$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 + x + h - x^2 - x}{h}$$

$$= \lim_{h \to 0} \frac{2hx + h^2 + h}{h} \quad 4$$

$$= \lim_{h \to 0} \frac{h(2x + h + 1)}{h}$$

$$= \lim_{h \to 0} 2x + h + 1 = 2x + 1$$

3

4. (10pts) Find the equation of the tangent line to the function $f(x) = x^3 - x$ at the point $x = 1$.

$f'(x) = 3x^2 - 1$ 2

$f'(1) = 2$ = slope of tangent line at $x = 1$. 2

$f(1) = 0$ 2

$y = 2(x-1) = 2x - 2$

4

5. (10pts) Find the slope of the tangent line to the curve determined by the equation

$$x^2y^2 + 4xy = 12y$$

at the point $(2, 1)$.

Differentiate (w.r.t. x) the expression

$2xy^2 + 2x^2yy' + 4y + 4xy' = 12y'$ (4)

Plug in $(x,y) = (2,1)$

$4 + 8y' + 4 + 8y' = 12y' \Rightarrow 8 = -4y' \Rightarrow y' = -2$

(4) (2)

6. (10pts) Examine the provided graph of the function $f(x)$ in Figure 1 and determine whether the following statements are true (**T**) or false (**F**) by entering the letter in the blank provided.

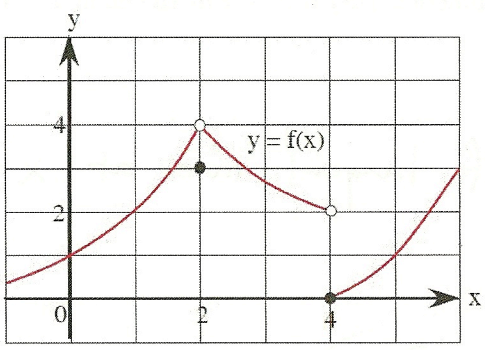


Figure 1

(a) F $f(2) = 4$. (2)

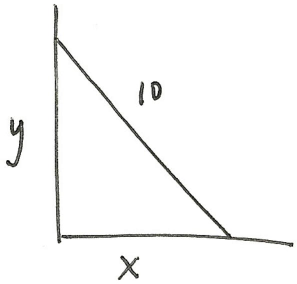
(b) T $\lim_{x\to 2} f(x) = 4$. (2)

(c) F f is continuous at $x = 2$. (2)

(d) T $f'(x) < 0$ when $2 < x < 4$. (2)

(e) F f is differentiable at $x = 2$. (2)

7. (10pts) A 10-foot tall ladder is leaning up against a building. If the bottom of the ladder is sliding away from the building at a rate of 1 foot per second, how fast is the top of the ladder sliding down the wall when the base of the ladder is 6 feet away from the wall?



$$x^2 + y^2 = 100$$ 4

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$ 2

$\frac{dx}{dt} = 1$ $\quad x = 6$ $\quad$ 2 $\quad y = \sqrt{100-6^2} = \sqrt{64} = 8$

So solve for $\frac{dy}{dt}$

$$2(6)(1) + 2(8)\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -3/4 \text{ ft/s}$$

2

8. (10pts) Consider the function $f(x) = \sqrt{x}$

(a) Find the linear approximation to the function $f(x) = \sqrt{x}$ at $x = 4$.

$f'(x) = \frac{1}{2}x^{-1/2}$ 2 $\Rightarrow f'(4) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$ 1

$f(4) = 2.$ 2

8

$$L(x) = 2 + \frac{1}{4}(x-4)$$

3

(b) Use your answer above to estimate $\sqrt{1.95}$.

$$\sqrt{1.95} \approx L(1.95) = 2 + \frac{1}{4}(1.95-4)$$

2