1210-90 Exam 1

Summer 2014

Name KEY

Instructions. Show all work and include appropriate explanations when space is provided. Correct answers unaccompanied by work may not receive full credit. Please circle your final answers.

1. (22pts) Let

$$f(x) = \frac{4x-4}{x^2-4x+3} = \frac{4(x-1)}{(x-1)(x-3)}$$

Answers below may be values, DNE (does not exist), or $\pm\infty$. You must show your work.

(a) (4pts) Compute $\lim_{x\to 3^+} f(x)$

4

When we plug in $x=3$, the numerator is nonzero but denominator is zero. So we have a vertical asymptote at $x=3$. When $x>3$,

$$\frac{4(x-1)}{(x-1)(x-3)} = \frac{+}{+\cdot+} > 0. \Rightarrow \lim_{x\to 3^+} f(x) = +\infty$$

(b) (4pts) Compute $\lim_{x\to 3^-} f(x)$

4

When $x<3$,

$$\frac{4(x-1)}{(x-1)(x-3)} = \frac{+}{+\cdot -} < 0 \Rightarrow \lim_{x\to 3^-} f(x) = -\infty$$

(c) (4pts) Compute $\lim_{x\to 3} f(x)$

4

Since $\lim_{x\to 3^+} f(x) \neq \lim_{x\to 3^-} f(x)$, $\lim_{x\to 3} f(x)$ DNE

(d) (4pts) Compute $\lim_{x\to+\infty} f(x)$

4

$$\lim_{x\to\infty} \frac{4x-4}{x^2-4x+3} = \lim_{x\to\infty} \frac{x(4-4/x)}{x^2(1-\frac{4}{x}+\frac{3}{x^2})} = \lim_{x\to\infty}\left(\frac{1}{x}\right) \lim_{x\to\infty} \frac{4-4/x}{1-\frac{4}{x}+\frac{3}{x^2}}$$

$$= 0\cdot 4 = 0$$

2 (e) (2pts) $f(x)$ has a horizontal asymptote at $y =$ 0.

2 (f) (2pts) $f(x)$ has a vertical asymptote at $x =$ 3.

2 (g) (2pts) $f(x)$ is continuous everywhere except $x =$ 1, 3 (list *all* values).

22

2. (16pts) Compute the following limits. Be sure to show your work.

(a) (4pts) $\lim_{x \to 0} \frac{x^2 - 2}{5 \cos x}$ $= \frac{0^2 - 2}{5 \cos 0} = \frac{-2}{5}$

since $5\cos(0) \neq 0$.

4

(b) (4pts) $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$ $= \lim_{x \to 2} \frac{(x-2)(x+3)}{x-2} = \lim_{x \to 2} x+3 = 5$

4

(c) (4pts) $\lim_{x \to 0} \frac{\sin x \cos x}{x}$ $= \lim_{x \to 0} \left(\frac{\sin x}{x}\right) \cdot \lim_{x \to 0} \cos x$

$= 1 \cdot 1 = 1$

4

(d) (4pts) $\lim_{x \to 0} \frac{(3 + x)^2 - 9}{x}$ $= \lim_{x \to 0} \frac{9 + 6x + x^2 - 9}{x} = \lim_{x \to 0} \frac{6x + x^2}{x}$

$= \lim_{x \to 0} 6 + x = 6$

4

3. (10pts) Use **the definition of the derivative** to compute the derivative of $f(x) = \sqrt{x}$; that is, compute the limit

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Hint: Use the algebraic identity $\frac{\sqrt{a} - \sqrt{b}}{c} = \left(\frac{\sqrt{a} - \sqrt{b}}{c}\right)\left(\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} + \sqrt{b}}\right) = \frac{a - b}{c(\sqrt{a} + \sqrt{b})}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

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$$= \lim_{h \to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}$$ (using identity)

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.$$

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4. (20pts) Compute the following derivatives using the derivative rules. There is no need to simplify.

(a) (5pts) $D_x(4x^5 + x^3 - 2x^2 + 9)$

5 $= 20x^4 + 3x^2 - 4x$

(b) (5pts) $D_x(\sin x \cos x)$

5 $= \cos^2 x - \sin^2 x$

(c) (5pts) $D_x(\frac{x^2}{1-x^3})$

5 $= \frac{(1-x^3)(2x) - (x^2)(-3x^2)}{(1-x^3)^2} = \frac{2x - 2x^4 + 3x^4}{(1-x^3)^2} = \frac{2x + x^4}{(1-x^3)^2}$

(d) (5pts) $D_x(\cos(x^5 + x))$

5 $= -\sin(x^5+x)(5x^4+1)$

5. (6pts) Suppose f and g are two functions whose values, and the values of their derivatives, are given by the following chart

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	1	3	-1	-2
1	2	4	2	1
2	-4	2	1	2

For example, $f(0) = 1$, $g'(2) = 2$, etc. Use derivative rules to fill in the blanks:

2 If $F(x) = f(x)g(x)$, then $F'(1) =$ $f'(1)g(1) + f(1)g'(1) = (4)(2) + (2)(1) = 10$

2 If $G(x) = \frac{f(x)}{g(x)}$, then $G'(1) =$ $\frac{g(1)f'(1) - f(1)g'(1)}{g(1)^2} = \frac{(2)(4) - (2)(1)}{(2)^2} = \frac{6}{4} = \frac{3}{2}$

2 If $H(x) = f(g(x))$, then $H'(1) =$ $f'(g(1))g'(1) = f'(2)g'(1) = (2)(1) = 2$

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6. (12pts) Consider the function $f(x) = 2 + \frac{x}{1+x^2}$.

(a) (4pts) Find $f'(x)$.

4 $\quad f'(x) = \dfrac{(1+x^2)(1) - (x)(2x)}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2}$

(b) (4pts) Find the equation of the tangent line to the graph of $y = f(x)$ at $x = 0$.

$f(0) = 2 \qquad f'(0) = 1$ $\qquad y = f(a) + f'(a)(x-a)$

4 $\quad y = 2 + 1(x-0) = 2 + x.$

(c) (4pts) At what points x is the tangent line to the graph of $y = f(x)$ horizontal?

4 $\quad 0 = f'(x) = \dfrac{1-x^2}{(1+x^2)^2} = \dfrac{(1+x)(1-x)}{(1+x^2)^2} \Rightarrow x = \pm 1$

7. (9pts) An object moves along a horizontal coordinate line so that its position (in meters) at time t (measured in seconds) is given by

$$s(t) = (t-1)^3 - 12t + 6.$$

(a) (3pts) Find the velocity of the object at time t.

3 $\quad v(t) = s'(t) = 3(t-1)^2 - 12 = 3t^2 - 6t - 9$

$= 3(t^2 - 2t - 3)$

(b) (3pts) Find the acceleration of the object at time t.

3 $\quad a(t) = 6t - 6$

(c) (3pts) At what time is the object the farthest to the left? In other words, when is the object's position the smallest (i.e. the most negative)?

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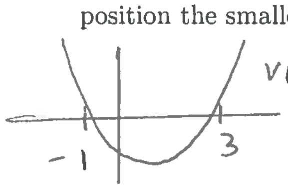


velocity is negative until $t = 3$, then becomes positive

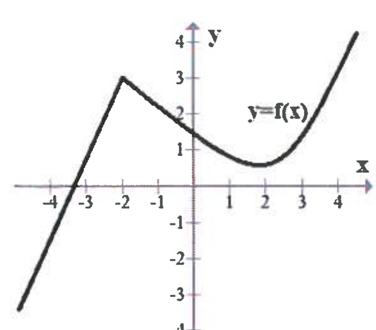
$t = 3$

8. (5pts) The graph of $y = f(x)$ is given below. Use it to answer the following questions:

2 (a) ___T___ True (T) or False (F): $f(x)$ is continuous on $(-4, 4)$.

2 (b) ___F___ True (T) or False (F): $f'(1) > f'(3)$.

1 (c) $f(x)$ is not differentiable at $x =$ ___-2___.



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