

1210-90 Exam 2
Summer 2013

Name _____

Instructions. Show all work and include appropriate explanations when necessary. A correct answer unaccompanied by work may not receive full credit. Please try to do all work in the space provided. Please circle your final answers.

1. (25pts) For this problem, consider the function

$$f(x) = 3x^5 - 5x^3 + 2.$$

(a) (4pts) Find $f'(x)$.

(b) (4pts) Find the three critical points of $f(x)$.

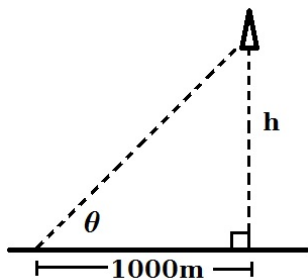
(c) (4pts) Use the First Derivative Test to determine whether each critical point is a local minimum, a local maximum, or neither.

(d) (4pts) Find $f''(x)$.

(e) (4pts) Find the inflection point(s) of $f(x)$, or show that there are no inflection points. It is acceptable to only list the x -values of the inflection points.

(f) (5pts) Find the minimum and maximum **values** of $f(x)$ on the interval $[-1, 2]$.

2. (8pts) A rocket launch is being watched by an observer 1000 meters away from the launch pad. When the rocket's height is 1000 meters, the observed angle of the rocket (labeled θ in the picture below) is increasing at a rate of $\frac{1}{20}$ radians per second. How fast is the rocket traveling at this time?



3. (5pts) Find the slope of the tangent line to the following curve at the point $(1, 2)$.

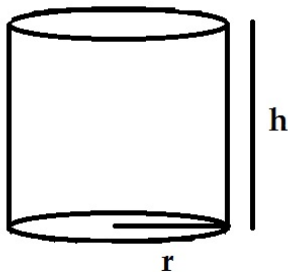
$$x^3 - 2xy^2 + y^3 = 1$$

4. (a) (4pts) Find the equation of the tangent line to the graph of $y = \sqrt{x}$ at the point $(16, 4)$.

(b) (4pts) Use part (a) above to estimate $\sqrt{16.2}$.

5. (5pts) Find the value c guaranteed by the Mean Value Theorem for $f(x) = x^3 - 2x$ on the interval $[0, 3]$

6. (8pts) A closed cylindrical can is being manufactured to hold 250π cm³ of liquid. What are the radius and height (labeled r and h in the picture below) of the can which can be made using the least amount of material? Minimize the surface area of the can $A = 2\pi r^2 + 2\pi rh$ subject to a fixed volume $V = \pi r^2 h$. **Note:** You must use calculus to get credit!!



7. (16pts) Find the indicated antiderivatives:

(a) (4pts) $\int (2x - 3) dx$

(b) (4pts) $\int \sqrt[5]{x} dx$

(c) (4pts) $\int (x^{-3} + \sin x) dx$

(d) (4pts) $\int (x^3 + 2x)^5 (3x^2 + 2) dx$

8. (a) (5pts) Approximate $\int_{-2}^3 |x| dx$ using a Riemann sum with 5 subintervals of equal length and the sample points being the right-endpoints of the subintervals.

- (b) (5pts) Find the exact value of $\int_{-2}^3 |x| dx$. **Hint:** Use geometry and the area interpretation of the definite integral. What does the graph of $y = |x|$ look like?

9. (10pts) The pull of the Earth's gravity is slightly less on the top of a tall building than it is on the Earth's surface. Suppose we drop a ball from the top of the Sears Tower (1730 feet). We estimate that, as the ball drops, it experiences an acceleration of

$$a(t) = -31 - \frac{t}{10}$$

feet per second squared. The negative sign indicates that gravity acts downward.

- (a) (5pts) Find $v(t)$, the velocity of the ball in feet per second after t seconds.

- (b) (5pts) Find an equation whose solution will give the time it takes for the ball to hit the ground. Do not attempt to solve.

10. (5pts) Compute x_2 , the second approximation to the root of

$$x^3 - 2 = 0$$

using Newton's Method with initial approximation $x_1 = 1$.