Calculus I
Exam 1, Fall 2002, Answers

1. Find the equation of the line which goes through the point (0,7) and is perpendicular to the line given by the equation $2x+3y=10$.

Answer. The given equation can be written as $y=-(2/3)x+10/3$. This line has slope $-2/3$, so the line we seek has slope $3/2$. Then, by the point-slope formula

$$\frac{y-7}{x-0}=\frac{3}{2}\ ,$$

which simplifies to $3x-2y=-14$.

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2. Find the derivatives of the following functions:

a) $f(x)=8x^3+3x^2-\dfrac{1}{x}=8x^3+3x^2-x^{-1}$

Answer. $f'(x)=24x^2+6x-(-1)x^{-2}=24x^2+6x+\dfrac{1}{x^2}$.

b) $g(x)=\dfrac{2x+5}{x-1}$

Answer. $g'(x)=\dfrac{(x-1)(2)-(2x+5)}{(x-1)^2}=\dfrac{-7}{(x-1)^2}$.

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3. Find the derivatives of the following functions:

a) $f(x)=(\sin(2x)+\cos(5x))^2$

Answer. $f'(x)=2(\sin(2x)+\cos(5x))(2\cos(2x)-5\sin(5x))$.

b) $g(x)=(1-x^2)^{15}$

Answer. $g'(x)=(1-x^2)^{14}(-2x)=-2x(1-x^2)^{14}$.

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4. Find the equation of the line tangent to the curve $y=x^3-x^2+1$ at (2,5).

Answer. The slope of the tangent line at (x,y) is $dy/dx=3x^2-2x$. At $x=2$, the value is $3(2)^2-2(2)=8$. Thus the equation is

$$\frac{y-5}{x-2}=8 \qquad \text{or} \qquad y=8x-11\ .$$

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5. A body is fallng toward the surface of the earth. Let $s(t),\ v(t)$ represent the distance fallen and the velocity of the object (relative to its position at time $t=0$, where the direction of increasing s is downward) at time t. Then we have the formula

$$s(t)=16t^2+v(0)t\ ,$$

If the velocity at time $t=0$ is 12 ft/sec, at what time will the object have a velocity of 100 ft/sec?

Answer. From the hypotheses, $v(0) = 12$, so the equation of motion is $s(t) = 32t + 12t$. Then

$$v(t) = \frac{ds}{dt} = 32t + 12 \; .$$

The velocity is 100 ft/sec at the time t for which $100 = 32t + 12$. Thus $t = 88/32 = 11/4$ seconds.