Calculus I, Mathematics 1210-90

Examination 1, February 12,14, 2004

1. Find the value of x where the graphs of these two functions have parallel tangent lines:

$$f(x) = x^2 - 3x + 2 \ , \qquad g(x) = 2x^2 - 11x - 17 \ .$$

Solution. Two lines are tangent if they have the same slope. We find the slope of the tangent lines by differentiating: $f'(x) = 2x - 3,\ g'(x) = 10x - 11$. So the two graphs have parallel tangent lines at the points where $f'(x) - g'(x)$. We solve:

$$2x - 3 = 10x - 11 \quad \text{or} \quad 8x = 8 \quad \text{or} \quad x = 1 \ .$$

2. Find the derivatives of the following functions:

a) $$f(x) = (x+1)(\frac{1}{x} + 1)$$

Solution. First write the function in exponential notation: $f(x) = (x+1)(x^{-1}+1)$ and then use the product rule:

$$f'(x) = (1)(x^{-1}+1) + (x+1)(-x^{-2}) = x^{-1} + 1 - x^{-1} - x^{-2} = 1 - x^{-2} \ .$$

b) $$g(x) = (\tan(3x) - 1)^2$$

Solution. Use the chain rule:

$$g'(x) = 2(\tan(3x) - 1)(\sec^2(3x))(3) = 6(\tan(3x) - 1)(\sec^2(3x)) \ .$$

3. Find the slope of the line tangent to the curve

$$y = x^2 - 3x + 1/x$$

at the point (3,1/3).

Solution. The slope of the tangent line is the value of the derivative at the point $x = 3$. Let $f(x) = x^2 - 3x + x^{-1}$. Then

$$f'(x) = 2x - 3 - x^{-2} \quad \text{so that the slope is} \quad f'(3) = 2(3) - 3 - 3^{-2} = \frac{26}{9} \ .$$

4. Let $y = x^3 - 48x + 1$. Find the x coordinate of the points at which the graph has a horizontal tangent line.

Solution. The graph has a horizontal line where $y' = 0$. Differentiating: $y' = 3x^2 - 48$, and solving $3x^2 - 48 = 0$ we find $x = \pm 4$.

2. On the planet Garbanzo in the Weirdoxus solar system, the equation of motion of a falling body is

$$s = s_0 + v_0 t - 10t^3$$

where s_0 is the initial height above ground level and v_0 is the initial velocity. Distance is measured in garbanzofeet. If a ball is thrown upwards from ground level at an initial velocity of 120 garbanzofeet/second, how high does the ball rise?

Solution. We are given $s_0 = 0$, $v_0 = 120$, so the equation of motion is $s = 120t - 10t^3$. Differentiating we get the equation for velocity: $v = 120 - 30t^2$. At the height of the motion the velocity is 0, so we have $0 = 120 - 30t^2$, so the ball is at its maximum height in $t = 2$ seconds. At this value of t, $s = 120(2) - 10(2)^3 = 160$ garbanzofeet.