Calculus I
Exam 3, Summer 2002, Answers

1. Integrate:

a) $\int (x^3+3x+5)^3(x^2+1)dx =$

Answer. Let $u = x^3+3x+5,\ du = 3(x^2+1)dx$. Then

$$\int (x^3+3x+5)^3(x^2+1)dx = \frac{1}{3}\int u^3 du = \frac{1}{12}(x^3+3x+5)^4 + C\ .$$

b) $\int (\sin^2 x+1)\cos x dx =$

Answer. Let $u = \sin x,\ du = \cos x dx$. Then

$$\int (\sin^2 x+1)\cos x dx = \int (u^2+1)du = \frac{1}{3}u^3 + u + C = \frac{1}{3}\sin^3 x + \sin x + C\ .$$

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2. Solve the differential equation:

$$\frac{dy}{dx} = (1+x)y^2, \quad y(1) = 2.$$

Answer. As an equation of differentials, this becomes $y^{-2}dy = (1+x)dx$. Integrating:

$$-\frac{1}{y} = x + \frac{x^2}{2} + C\ .$$

Putting in the initial values we get $-1/2 = 1+1/2+C$, so $C = -2$. Putting in this value of C, and solving for y:

$$y = (2 - x - \frac{x^2}{2})^{-1}\ .$$

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3. Calculate the definite integrals:

a) $\int_0^4 (x^3+3x+1)dx$

Answer.

$$\int_0^4 (x^3+3x+1)dx = (\frac{x^4}{4} + \frac{3x^2}{2} + x)\Big|_0^4 = 64+24+4 = 92\ .$$

b) $\int_0^{\pi/2} (\sin x\cos x)dx$

Answer.

$$\int_0^{\pi/2} (\sin x \cos x)dx = \frac{\sin^2 x}{2}\Big|_0^{\pi/2} = \frac{1}{2}\ .$$

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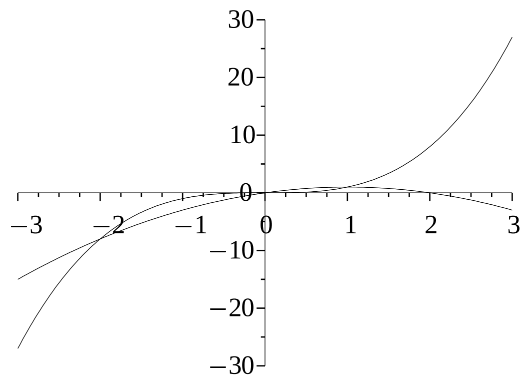
4. Find the area of the region in the third quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$.

Answer. Draw the graph. The region in question is that in the third quadrant running from $x = -2$ to $x = 0$. Since the upper curve is $y = x^3$, the area is

$$\int_{-2}^0 x^3 - (2x - x^2)dx = \frac{x^4}{4} - x^2 + \frac{x^3}{3}\Big|_{-2}^0 = -[\frac{16}{4} - 4 - \frac{8}{3}] = \frac{8}{3}\ .$$

PSfrag replacements



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5. The region in the first quadrant bounded by the curves $y = \sqrt{x}$ and $y = x$ is rotated about the y-axis. What is the volume of the solid so produced?

Answer. Draw the graph. Sweeping along the x axis and using the shell method we obtain

PSfrag replacements

$$V = \int_0^1 2\pi x(\sqrt{x} - x)dx = 2\pi \int_0^1 (x^{3/2} - x^2)dx = 2\pi(\frac{2}{5} - \frac{1}{3}) = \frac{2\pi}{15} .$$

Sweeping along the y-axis and using the washer method we first rewrite the curves as $x = y$ and $x = y^2$. Then

$$V = \int_0^1 (\pi y^2 - \pi y^4)dy = \pi \int_0^1 (y^2 - y^4)dy = \pi(\frac{1}{3} - \frac{1}{5}) = \frac{2\pi}{15} .$$