

**Mathematics 1210-90**  
**Final Examination December 10,11, 2003**

**You must show your work. Just entering an answer will earn no points.**

1. Let  $f(x) = \frac{x^2}{1+x^2}$

a)  $f'(x) =$

b)  $f''(x) =$

2. Integrate:

a)  $\int (4x^2 + x - x^{-2})dx =$

b)  $\int \frac{\sqrt{x}}{(1+x^{3/2})^2} dx =$

3. The volume of a cone of radius  $r$  and height  $h$  is  $V = \frac{\pi}{3}r^2h$ . Water is pouring into a conical cup of radius 8 cm and height 10 cm at the rate of 120 cm<sup>3</sup>/min. At what rate is the height of water in the cup rising when it is at  $h = 5$  cm (and  $r = 4$ )?

4 a). Graph  $y = 2x + \frac{1}{x}$  for  $x > 0$ .

b) What is the minimum value of  $y$ ?

5. Find the solution to the differential equation

$$\frac{dy}{dx} = \frac{3x}{y+1}$$

such that  $y(0) = 4$ .

6. A curve in the plane is given by the equation  $x^3 - y^3 = 61$ . What is the slope of the tangent line to the curve at the point (5,4)?

7. Find the area of the region in the first quadrant bounded by the curve  $y = 9x - x^2$ .

8. The region in the first quadrant bounded by the curves  $y = 9x$  and  $y = x^3$  is rotated about the  $y$ -axis. Find the volume of the resulting solid.

9. What is the center of mass of the triangle bounded by the coordinate axes and the line  $2x + y = 1$ ?

10. Find  $\int_0^{\pi/2} \sqrt{\sin x \cos x} dx$