MATH 1210-90 Fall 2011

Third Midterm Exam

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LAST NAME ______________________

FIRST NAME Grader's Copy

ID NO. ______________________

INSTRUCTION: SHOW ALL OF YOUR WORK. MAKE SURE YOUR ANSWERS ARE CLEAR AND LEGIBLE. USE **SPECIFIED** METHOD TO SOLVE THE QUESTION. IT IS NOT NECESSARY TO SIMPLIFY YOUR FINAL ANSWERS.

PROBLEM 1	40	______
PROBLEM 2	20	______
PROBLEM 3	20	______
PROBLEM 4	20	______
TOTAL	100	______

PROBLEM 1

(40 pt) Analyze the funtion.

$$y = f(x) = \frac{x}{1+x^2}.$$

(3pt) each

(1) Domain and range.

Domain : all real numbers

~~range~~ : $[-1, 1]$ ← hard to determine known till sketch

(2pt)

(2) Symmetry.

$f(-x) = -f(x)$ odd

(3) x- and y-intercepts.

x : (0, 0) y : (0, 0)

(4) Find the first derivative of f.

$$f'(x) = \frac{1-x^2}{(1+x^2)^2}$$

→ quotient rule

(5) Find the second derivative of f.

$$f''(x) = \frac{(-2x)(3-x^2)}{(1+x^2)^3}$$

(6) Find the critical points, if any.

$1 - x^2 = 0 \qquad x = \pm 1$

(7) Find the inflection points, if any.

$x = 0 \quad \text{or} \quad x = \pm\sqrt{3}$

(8) Find the intervals where f is increasing, and the intervals f is decreasing.

inc : $(-1, 1)$

dec : $(-\infty, -1) \cup (1, \infty)$

$- \quad + \quad -$
$-1 \qquad 1$

(9) Find the intervals where f is concave up, and the intervals f is concave down.

up : $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

down : $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$

$- \quad + \quad - \quad +$
$-\sqrt{3} \qquad 0 \qquad +\sqrt{3}$

(10) Find the asymptotes.

No vertical asym.

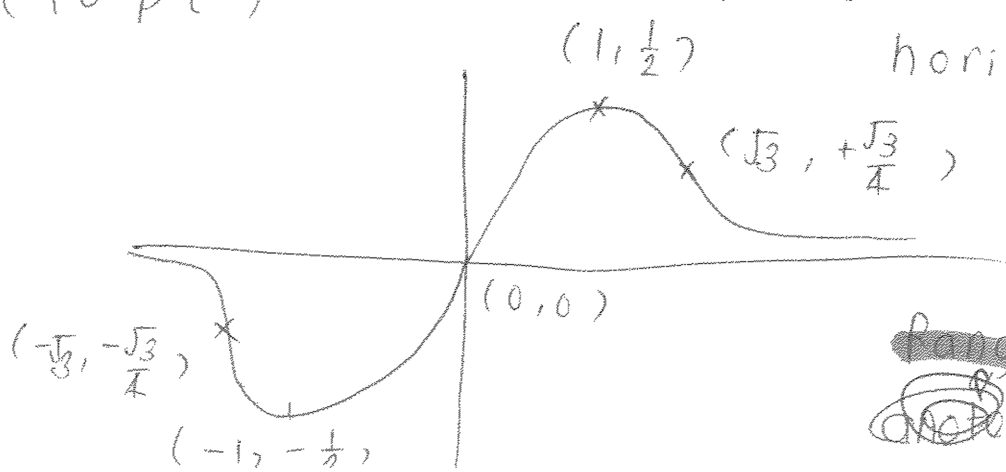
$\lim_{x \to \infty} f(x) = 0$ horizontal 1 : $y = 0$

$\lim_{x \to -\infty} f(x) = 0$

horizontal 2 : $y = 0$

Sketch the graph of f.

(10 pt)

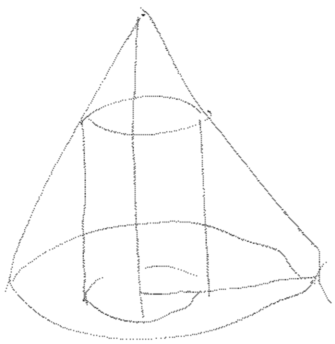


Range : $[-1, 1]$!!

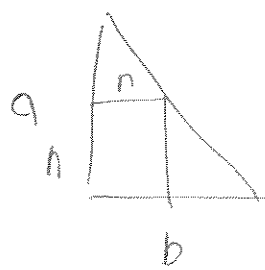
~~another ans~~ (2pt)

PROBLEM 2

(20 pt) Find the dimension of the right circular cylinder of greatest volume that can be inscribed in a give right circular cone.
Hint: Let a be the altitude and b be the radius of the base of the given cone. Find out the altitude, radius, and volume, respectively, of an inscribed cylinder.



Similar triangle



(5 pt)

$$\frac{a}{b} = \frac{a-h}{r}$$

$$r = b\left(1 - \frac{h}{a}\right) \quad (5\ pt)$$

$$V = \left[b\left(1 - \frac{1}{a} h\right)\right]^2 \cdot h \cdot \pi$$

$$V = b^2 \pi \left(1 - \frac{h}{a}\right)^2 h$$

(5 pt) $$\frac{dV}{dh} = \pi b^2 \left(1 - \frac{h}{a}\right)\left(1 - \frac{3}{a} h\right) = 0$$

$h \neq a$ $(r = 0)$ (meaningless)

$$h = \frac{1}{3} a \qquad r = \frac{2}{3} b \qquad V = \pi \cdot \left(\frac{3}{2}\right)^2 b \left(\frac{1}{3} a\right)$$

(5 pt)

PROBLEM 3

(20 pt) Use Newton's method to find an approximation solution to the equation

$$x^3 + x = -3$$

as follows. Let $x_1 = -1$ be the initial approximation. What is the second approximation x_2?

$y = x^3 + x + 3$

(5 pt)

$y' = 3x^2 + 1$

tangent line:

through $(-1, 1)$ (5 pt)

$m = 4$

$y = 4(x+1) + 1$ (5 pt)

$= 4x + 5 = 0$

$x_2 = -\frac{5}{4}$ (5 pt)

PROBLEM 4

(20 pt) Consider the differential equation:

$$\frac{du}{dt} = -u^2(t^3 - t).$$

Find the particular solution of the above differential equation that satisfies the condition $u = 4$ at $t = 0$.

$$-u^{-2}\frac{du}{dt} = t^3 - t \quad (5pt)$$

$$\int -u^{-2}\, du = \int t^3 - t \; dt \quad (5pt)$$

$$u^{-1} = \frac{1}{4}t^4 - \frac{1}{2}t^2 + C$$

(5pt)

$$\frac{1}{4} = C$$

(5pt)

Sol: $$u^{-1} = \frac{1}{4}t^4 - \frac{1}{2}t^2 + \frac{1}{4}$$

or $$u = \left[\frac{1}{4}t^4 - \frac{1}{2}t^2 + \frac{1}{4}\right]^{-1}$$