Solutions for Introduction to Polynomial Calculus

Section 5 Problems - Definite Integrals of Polynomials

Bob Palais

Calling the function in each problem $f(x)$ and using the three antidifferentiation rules of the previous section and the result that $\int_a^b f(x)dx = F(b) - F(a)$, where F is any antiderivative of f (we will often use the shorthand $|_a^b F(x)$ for $F(b) - F(a)$):

(1) $F(x) = \int f(x)dx = \frac{x^3}{3} - x^2 + x + C$, and choosing $C = 0$ for simplicity,

$|_1^5 F = \frac{5^3}{3} - 5^2 + 5 - (\frac{1^3}{3} - 1^2 + 1) = 64/3 = 21\frac{1}{3}$.

(2) $F(x) = \int f(x)dx = \frac{x^4}{4} + 2x + C$, $|_0^2 F = \frac{2^4}{4} + 4 - (\frac{0^4}{4} + 0) = 8$.

(3) $F(x) = \int f(x)dx = \frac{x^5}{5} - \frac{x^6}{6} + C$, $|_0^1 F = \frac{1^5}{5} - \frac{1^6}{6} - (\frac{0^5}{5} - \frac{0^6}{6}) = 1/5 - 1/6 = 1/30$.

(4) $F(x) = \int f(x)dx = \frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2} + C$,

$|_0^1 F == \frac{1^{n+1}}{n+1} - \frac{1^{n+2}}{n+2} - (\frac{0^{n+1}}{n+1} - \frac{0^{n+2}}{n+2}) = \frac{1}{n+1} - \frac{1}{n+2} = \frac{1}{(n+1)(n+2)}$.

Using the fact that the area under the curve $y = f(x)$ and above the x-axis from $x = a$ to $x = b$ is given by $\int_a^b f(x)dx$, the area under the curve $y = f(x) = x^2 + 5x$ from $x = 3$ to $x = 4$ is $\int_3^4 f(x)dx$. Then as in the first four problems,

(5) $F(x) = \int f(x)dx = x^3 + x^2 + x + C$, $|_1^2 F = 2^3 + 2^2 + 2 - (1^3 + 1^2 + 1) = 11$.

(6) $F(x) = \int f(x)dx = \frac{x^3}{3} + 5\frac{x^2}{2} + C$, $|_3^4 F = \frac{4^3}{3} + 5\frac{4^2}{2} - (\frac{3^3}{3} + 5\frac{3^2}{2}) = 179/6$.

(7 old version) $F(x) = \int f(x)dx = \frac{x^{11}}{11} - \frac{x^{10}}{10} + C$, $|_1^3 F = \frac{3^{11}}{11} - \frac{3^{10}}{10} - (\frac{1}{11} - \frac{1}{10})$. We could factor 3^{10} from the first term, leaving $3^{10}(\frac{3}{11} - \frac{1}{10}) - (\frac{1}{11} - \frac{1}{10}) = 3^{10}(\frac{19}{110} - (\frac{-1}{110})$ or finally $\frac{(19 \cdot 3^{10} + 1)}{110}$.
(7 new version) $F(x) = \int f(x)dx = \frac{x^4}{4} - x^3 + C$, $|_1^3 F = \frac{3^4}{4} - 3^3 - (\frac{1}{4} - 1) = \frac{80}{4} - 27 + 1 = -6$.

(8) Rewriting the fundamental theorem of calculus as $F(b) = F(a) + \int_a^b f(t)dt$ where $F' = f$ so is any antiderivative of f, and observe that the (horizontal) displacement is the antiderivative of the horizontal velocity. So

$$s(3) = s(1) + \int_1^3 2t + 3t^2 + 1dt = s(1) + |_1^3 t^2 + t^3 + t = s(1) + (39 - 3)$$

So at time $t = 3$, the particle is at the point $s(3) = 36$ feet to the right of its position at $t = 1$ (the origin).

Alternatively, solve for $s(t)$ explicitly:

$$s(t) = \int v(t)dt = \int 2t + 3t^2 + 1dt = t^2 + t^3 + t + C$$

feet. We are given that $s(1) = 3 + C = 0$ feet, so $C = -3$ and $s(t) = t^2 + t^3 + t - 3$, and plug in to get $s(3) = 36$.