

Diagnostic Test in Algebra and Trigonometry

This test comes in two parts. The first part tests whether or not you really know what your calculus instructor assumes you know. These are typical problems from courses in Algebra and Trigonometry. If you get less than 9 problems correct on the first try, your background could be an obstacle to performing well in Calculus. If you cannot do at least half the problems, you will have great difficulty in Calculus, and shall have to restudy algebra intensively. In any case, do the entire test thoroughly, making sure that you know how to do every one of these problems.

The second part consists of actual algebra and trigonometry problems which come up in solving specific problems in Calculus. Typically, to solve a Calculus problem, you use a principle or idea of calculus to set up a problem in algebra, which you then solve. The problems in part II are the algebraic (or trigonometric) part of problems which have appeared on calculus exams.

You will note that answers are provided in an accompanying document. Besides the answer, some attempt is made to show how the problems are solved. Be fair to yourself: do not consult the answers until you have finished going through the problems.

Part I

1. Solve: $x - 2(3x - 1) = 7(3 - 2x)$.

2. What is the equation of the line through the points (2,1) and (7,-5)?

3. Simplify: $\frac{5 - \sqrt{3}}{4 + \sqrt{3}}$

4. Solve

$$x + 2y = 11$$

$$8x - 3y = 31$$

5. Solve: $x^2 + 3x = 2(1 - x^2)$.

6. Solve

$$\frac{1}{1+x} + \frac{1}{1-x} = \frac{9}{4}$$

7. Let

$$f(u) = \frac{u+1}{u^2+1}$$

For $u(x) = 2x - 1$, what is $f(u(x))$?

8. Let

$$y = \frac{1-x}{1+x}$$

Write x in terms of y .

9. Solve: $\sqrt{2x-1} = (x+1)/2 - 6$

10. If the hypotenuse of a right triangle is of length 5 and one leg is of length 2, what is the length of the other leg?

11. If $\sin \theta = 3/5$, what is $\cos \theta$?

12. Verify the identity:

$$\frac{\cos x + \sin x}{\cos x - \sin x} = \frac{1 + \sin(2x)}{\cos(2x)}$$

Part II

1. $\frac{-2}{2/5} + \frac{3}{2/7} - \frac{1/2}{2/9} =$

2. Solve $\frac{2x(x+2)^{1/2} - (3x+1)(x+2)^{-1/2}}{x+2} = 0$

3. Solve $\sin x + 2 - \sin^2 x + \cos^2 x = 0$

4. Solve $\sqrt{x+5} + \sqrt{x-1} = 11$.

5. Solve for x and y : $(x-1)^2 = y-1$, $2x+y = 11$.

6. Find the equation of the line through (2,1) which is perpendicular to the line through the points (5,8) and (3,-1).

7. $\frac{x^3 + 10x^2 + 3x - 54}{x^2 + 7x - 18}$

is a polynomial. Why? Find the polynomial.

8. $\frac{x-7}{x+13} = 1 - \frac{a}{x+13}$. $a = ?$

9. For what values of x between 0 and 2π (angle measured in radians) is $\sin x > \cos x$?

10. Consider the expression

$$f(x) = \frac{x(x-1)}{(x+1)(x-2)}$$

- a) If $x < -1$, what is the sign of $f(x)$?
- b) If $-1 < x < 0$, what is the sign of $f(x)$?
- c) If $x > 2$, what is the sign of $f(x)$?
- d) How many times does the sign of $f(x)$ change as x ranges from -2 to 4?