Calculus I
Practice Problems 4

1. Find the the equation of the tangent line to the curve $y = x - x^{-2}$ at (2,7/4).

2. Differentiate: $y = (x^2 - 1)\sin(x^2 + 1)$.

3. Find $f'(x)$: $f(x) = \dfrac{(x+1)^2}{(x-1)^2}$

4. Find $g'(x), g''(x)$: $g(x) = (x^3 + 1)^4$.

5. Find the derivative: $h(x) = (\cos(2x) + 1)\sin(3x)$

6. Find the derivatives of the following functions:
   a) $f(x) = \cos^2 x$
   b) $g(x) = \dfrac{\sin^2 x}{\cos x}$

7. Find the first and second derivatives of $f(x) = x\sqrt{1 - x^2}$

8. Differentiate: $g(x) = (\sin(3x) + 1)^3$.

9. Differentiate: $h(t) = \dfrac{1 - t^2}{1 + t^3}$

10. Differentiate: $f(x) = \sqrt{2x^2 - 3x + 1}$.

11. Find the points on the curve $y = 3x^2 - 3x + 1$ whose tangent line is perpendicular to the line $x + 2y = 7$.

12. Consider the curves $C_1 : x^2 + y^2 = 1$, $C_2 : 2x^2 + y^2 = 2$ for y positive. For each x, the vertical line through $(x, 0)$ intersects the curves C_1, C_2 at the points (x, y_1), (x, y_2). Let $L(x)$ be the length of the line segment joining these two points. Find $L'(x)$.

13. Let $\mathscr{P}$ be an upward-opening parabola whose axis is the y-axis and whose vertex is the origin. Suppose the line $y = C$ intersects the parabola in two points. Show that the tangent lines at these points intersect on the line on the axis of the parabola (the y-axis).

14. Suppose that a point moves along the x-axis according to the formula $x(t) = 1/(t^2 + 1)$. Let A(t) be the area of the circle with diameter joining the origin to the point $x(t)$. Find $A'(t)$ when t=3.