Syllabus for Math 3210 - 3220 Foundations of Analysis I - II

Texts: Joe Taylor, *Foundations of Analysis*. Available for download at
`http://www.math.utah.edu/~taylor/foundations.html`
Anne Roberts, *Basic Logic Concepts*. Available for download at
`http://www.math.utah.edu/%7Earoberts/M3210-1d.pdf`

Math 3210-20 has two objectives, to cover the theory of one and several variable calculus and to train the student in essentials of the professional mathematician : logic, proof, and how to write a mathematical argument. Prerequisites are three semesters of calculus. Starting fall 2008, Math 2270 (linear algebra) will be a prerequisite for math 3220 and Math 2200 (finite math) will be strongly recommended for Math 3210. Students often take differential equations concurrently. So students have had very little exposure to proofs, programming or writing logically. Students who will be able to take finite math M2200 beginning fall 2008 should be better able to cope with the high demands of M3210. Still, the pacing of lectures should be slow and methodical at first, building the student's ability to handle proofs. Because some mathematics courses require Math 3210-20 as prerequisites, you are expected to cover all of the material listed for each semester. M3210 deals exclusively with one variable calculus; M3220 deals with several variables. This necessitates repetition of some material, although often in a more abstract version, *e.g.*, utilizing notions of Euclidean topology. I consider this pedagogically advantageous. Proofs that are patterned after the one variable case are often not repeated in the text, although these can be repeated as well. Taylor's notes give the student a clear and concise account of the material. They succeed and are organized similarly to the text by W. Wade, *An Introduction to Analysis,* 3rd ed., which is a useful reference.

Homework should be assigned regularly and graded conscientiously, giving students plenty of feedback. I suggest two or three midterms and final given in class each semester. The instructor should grade all exams. Students found optional extra problem sessions outside of regular class time in which they are given the chance to present solutions helpful. It is not recommended that students take M3210 and M3220 concurrently.

Math 3210 should cover Taylor's chapters 1-6. (This schedule is based on class meeting four days a week for 50 min.)Topics to be covered

Ch 0. Introduction to logic, sets, functions. (6 lectures.)

From Anne Roberts' notes and Taylor 1.1. Logic, quantifiers, sets. Alternatively, you could hand out introductory material (such as Ch. 1.1-2.7 of S. Lay, *Analysis with an Introduction to Proof*, 3rd ed. or the appendix of A. Mattuck, *Introduction to Analysis*, Prentice Hall, 1999.)

Ch 1. Real Numbers (10 lectures)

From Taylor Ch 1.2, 1.3. Omit the construction of real numbers via Dedekind Cuts of Ch. 1.4 and verification of properties of real numbers. (Real numbers are constructed in Math 5210.) Assume that the reals exist (Th. 1.4.5) and discuss ordered field properties, Archimidean property, well ordering and induction, completeness, (CF Wade, Ch. 1.) Ch. 1.5 sup/inf.

Ch 2. Real Sequences (8 lectures)

Taylor, Ch. 2.1-2.6. Limits of sequences, applications of limits, theorems about limits, monotone sequences, Cauchy Sequences, Bolzano Weierstrass Theorem, lim inf / lim sup.

Ch 3. Continuous Functions in R (8 lectures)
Continuity, composition, properties of continuous functions, Intermediate Value Theorem, inverse functions, uniform continuity, sequences of functions, uniform convergence, uniformly Cauchy sequences.

Ch 4. Derivative of Real Functions (9 lectures)
Taylor Ch 4.1-4.One-sided limits, infinite limits, difference quotient, differentiation theorems, chain rule, inverse function, mean value theorem, monotone functions, uniform continuity, Cauchy's mean value theorem, L'Hôpital's Rule.

Ch 5. Riemann Integral in R (8 lectures)
Taylor, Ch 5.1-5.4. Upper and lower sums, partitions, integral, existence of integrsl, properties of integral, Fundamental theorem of Calculus, substitution, integration by parts, log, exponential, improper integrals.

Ch 6. Infinite series (7 lectures)
Taylor, Ch 6.1-6.5. Infinite series, geometric series, series with nonnegative terms, comparison tests, integral test, root test, absolute and conditional convergence, products of series, power series, taylor's formula, lagrange's remainder.

Math 3220 should cover Taylor's chapters 7-10. Topics to be covered

Ch. 7 Convergence of sequences in R^n. (17 lectures)
Taylor Ch 7.1-5. Inner product, norm, distance, triangle inequality in R^n. Vector space, inner product space, Schwarz Inequality, normed vector spaces, e.g., C(I), metric spaces. Convergence of sequences in a metric space. Limit theorems. Cauchy sequences. R^n topology: open/closed sets, interior, closure, boundary. Compact sets, heine Borel theorem in R^n. Connected sets. It is better to limit the time spent on topology (which is covered in math 4510 anyway) to make sure you have enough time to cover integration theory of Ch. 10 thoroughly.

Ch. 8 Functions of Euclidean Space. (8 lectures)
Taylor Ch. 8.1-3. Continuous fuctions of several variables. Sequences and continuity. Parameterized surfaces. Continuous functions and open sets. Uniform continuity. Sequences and series of functions. Uniform convergence. Uniformly Cauchy. Weierstrass M-test. Sections 8.4-8.5, matrix theory and linear algebra can be skipped (or be done quickly as it is a review of Math 2280 material. I found reviewing concepts frome linear algebra as they are needed to be adequate.)

Ch. 9 Differentiation in Several Variables. (17 lectures)
Taylor, Ch 9.1-7. Fitzpatrick 17.4. Partial derivatives. Higher derivatives. Equality of mixed partials. Differential and linearization. Jacobian matrix. Chain rule. Change of variables. Directional derivatives. Tangent space of parameterized surface. Taylor's formula, mean value theorem. Necessary conditions for the maximum of a function. Inverse function theorem. Implicit function theorem. Level set as a parameterized surface. Cover also the necessary condition for the maximum of a function under equality constraint and Lagrange Multipliers. I covered section 17.4 in *Advanced Calculus 2nd ed.* by Patrick Fitzpatrick,

Brooks/Cole Pub., 2006. Alternatively do section 11.7 of Wade. (Because Taylor's notes have discussed the constraint set as a parameterized surface, it is possible to make a far more elegant presentation of this material than given in Fitzpatrick or Wade. The Lagrange multiplier formula follows directly from the fundamental theorem of linear algebra.)

Ch 10. Integration in Several Variables. (14 lectures)

Taylor Ch 10.1-10.5. Integration over a rectangle. Upper and lower sums. Upper and lower integrals. Properties of the integral. Jordan Regions. (Taylor defines a Jordan region as one whose characteristic function is integrable. Hence he does not need a separate discussion, as does Wade.) Properties of volume. Characterization of Jordan Regions. Integration over Jordan regions. Integrals of sequences. Iterated integrals. Fubini's Theorem. Iterated integrals over non-rectangular regions. The change of variables formula. Integral over a smooth image of a Jordan Region.

(Sample Syllabus--based on 3 lectures per week.)

Math 3210 Foundations of Analysis I

Fall Semester 2006

Instructor: David C. Dobson
Office: LCB 210
Office hours: MTF 12:50-1:40 or by appointment.
Prerequisites: Math 2210 or consent of instructor.

Text: *Foundations of Analysis,* by Joseph L. Taylor._**Supplemental notes:** *Basic Logic Concepts*, by Anne Roberts.

Homework: Problems will be assigned weekly and will generally be due in class on Wednesday of each week. Late homework cannot be accepted. A random subset of problems from each assignment will be graded. To succeed in the course, it is necessary to do and understand all of the homework.

Due date	Assignment
Wednesday 8/30	Problems 1-6 of Anne Robert's assignment 1.
Wednesday 9/06	Problems 1, 2 of Anne Robert's assignment 2, Exercise Set 1.1: 2, 3, 6, 8, 11, 15.
Wednesday 9/13	Exercise Set 1.2: 2, 4, 6, 9, 12, 13, 15, 16.
Wednesday 9/20	Exercise Set 1.3: 3, 6, 8, 10. Exercise Set 1.4: 1 (no proofs required), 3, 7, 10.
Wednesday 9/27	Exercise Set 1.5: 1, 3, 7, 12. Exercise Set 2.1: 2, 5, 6, 11.
Wednesday 10/4	(Exam I on Monday--reduced homework set!) Exercise Set 2.2: 2, 6, 10, 11.
Wednesday 10/11	Exercise Set 2.3: 4, 5, 9. Exercise Set 2.4: 3, 12. Exercise Set 2.5: 5, 7, 8. Exercise Set 2.6: 3, 6.
Wednesday 10/18	Exercise Set 3.1: 3, 5, 9, 11. Exercise Set 3.2: 1, 2, 4, 8.
Wednesday 10/25	Exercise Set 3.3: 1, 3, 5, 9. Exercise Set 3.4: 1, 4, 6, 8.
Wednesday 11/1	Exercise Set 4.1: 2, 5, 7, 10, 13, 15.
Wednesday 11/8	(Exam II on Monday--reduced homework set!) Exercise Set 4.2: 2, 5, 7, 11.
Wednesday 11/15	Exercise Set 4.3: 1, 4, 9, 10. Exercise Set 4.4: 6, 8, 10. Exercise Set 5.1: 1, 5, 8.
Wednesday 11/22	Exercise Set 5.2: 2, 3, 8, 9, 10. Exercise Set 5.3: 4, 5, 6, 9.
Wednesday 11/29	Exercise Set 5.4: 9, 10, 11, 13. Exercise Set 6.1: 2, 5, 9, 12. Exercise Set 6.2: 1, 2, 11.
Wednesday 12/6	Last homework! Exercise Set 6.3: 1, 3, 6, 7, 10.

	Exercise Set 6.4: 1, 4, 6.

Exams: Two midterm exams and a comprehensive final exam will be given in class. Exam I is closed-book, closed-notes. For Exam II, a single 8 1/2 x 11 sheet of notes is allowed. For the Final Exam, two 8 1/2 x 11 sheets of notes are allowed. No electronic devices are allowed.

Grades: Your grade will be determined by your scores on the three exams, and the total of your homework scores. The dates and weights of these are as follows:

Homework	(weekly)	40%
Exam I	(Monday, October 2)	15%
Exam II	(Monday, November 6)	20%
Final Exam	(Tuesday, December 12, 10:30 am-12:30 pm)	25%

There will be no opportunities for extra credit. Makeups for exams and homework will only be given in case of University-excused absences.

Tutoring is available from the *Math Tutoring Center*.

Students with disabilities may contact the instructor at the beginning of the semester to discuss special accomodations for the course.

(Sample Syllabus - - based on four lectures per week.)

Math 3220 §1 **Foundations of Analysis II** August 15, 2007

MTWF 9:40 - 10:30 in JTB 110.

Instructor: A. Treibergs, JWB 224, 581_8350
Office Hours: 10:45-11:45 MWF (tent.) & by appt.
E-mail: treiberg@math.utah.edu
Homepage: http://www.math.utah.edu/~treiberg/M3223.html

Text: Joseph L. Taylor, *Foundations of Analysis*, (2007)
PDF Notes available for download from
http://www.math.utah.edu/~taylor/foundations.html

Grading

Homework: To be assigned weekly.

Midterms: There will be three in-class one-hour midterm exams on Wednesdays Sept. 5, Oct. 3 and Nov. 7.

Final Exam: Tue., Dec. 11, 8:00-10:00 AM. Half of the final will be devoted to material covered after the third midterm exam. The other half will be comprehensive. Students must take the final to pass the course.

Course grade: Best two of three midterms 36% + homework 37% + final 27%.

Withdrawals: Last day to drop a class is Aug. 29. Last day to add a class is Sept. 4. Until Oct. 19 you can withdraw from the class with no approval at all. After that date you must petition your dean's office to be allowed to withdraw.

ADA: The Americans with Disability Act requires that reasonable accommodations be provided for students with cognitive, systemic, learning and psychiatric disabilities. Please contact me at the beginning of the quarter to discuss any such accommodations you may require for this course.

Objectives: To refine our skill at proof and facility with computation, to gain an appreciation for abstraction from the concepts of topology and metric spaces, and to learn the theory behind multidimensional calculus.

Topics: We shall try to cover the following chapters

Chapter 7.	Convergence in Euclidean Space
Chapter 8.	Functions on Euclidean Space
Chapter 9.	Differentiation in Several Variables
Chapter 10.	Integration in Several Variables

Math 3220 - 1 **Homework Problems** Fall 2007 Treibergs

You are responsible for knowing how to solve the following exercises from the text "Foundations of Analysis" by Joseph L. Taylor. Please hand in the starred "*" problems from the text and the additional exercises.

FIRST HOMEWORK ASSIGNMENT Due Monday, August 27, 2007.

7.1.1 [2, 3, 4*, 5*, 6*, 7, 9, 12*]

SECOND HOMEWORK ASSIGNMENT Due Tuesday, September 4, 2007.

7.2 [1*, 2, 3, 4, 5, 8*, 11*]

A*. Let (X,d) be a metric space. Suppose x, y are points in X and $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_n \to x$ and $y_n \to y$ as $n \to \infty$. Show that

$d(x_n,y_n) \to d(x,y)$ as $n \to \infty$.

B*. Let $\{u_n\}$ be a sequence in E^d and let u be a point in E^d. Suppose that for all vectors v in E^d we have $v \bullet u_n \to v \bullet u$ as $n \to \infty$. Show that $u_n \to u$ as $i \to \infty$.

THIRD HOMEWORK ASSIGNMENT Due Monday, September 10, 2007.

7.2 [12*, 13*, 16], 7.3 [1, 2]

A*. Let a, b and c be real numbers such that $a^2 + b^2 = 1$. Let $L = \{ (x,y) \in R^2 : ax + by = c\}$ be the points on a line in the plane. Show that L is a closed set.

B*. Let $S = U_{n=1}^{\infty} \{\boldsymbol{x}_n\}$ be the set in R^2 consisting of points from the sequence $\boldsymbol{x}_n = (1/n, 1/n)$. Determine whether S is open, closed or neither. Prove your answer.

FOURTH HOMEWORK ASSIGNMENT Due Monday, September 17, 2007.

7.3 [4*, 6, 8, 9, 10*, 12*], 7.4 [1*, 2, 3, 4*, 5]

A*. Let E be a subset of R. How many different sets can be obtained from E by taking closure or complementation? Prove your answer. If we denote closure by E^- and complement by E^c, then the question is: at most how many different sets can occur in the sequence ..., E^{-c-}, E^{-c}, E^-, E, E^c, E^{c-}, E^{c-c}, ... ?

B*. Show that every open set G in R^d is the union of at most countably many open balls

$G = U_{i=1}^{\infty} B(c_i, r_i)$.

[Hint. Consider balls whose centers have rational coordinates and whose radii are rational.]

FIFTH HOMEWORK ASSIGNMENT Due Friday, September 28, 2007.

7.5 [1, 2*, 3, 4, 6*, 11*, 12], 8.1 [1, 3*, 5*]

A*. Prove that the "topologist's sine curve" E in the plane is connected.

$E = \{ (0,y) : -1 < y < 1\} \cup \{ (x, \sin(1/x)) : 0 < x < 1\}$

B*. Suppose I and J are open intervals in the line and a is in I and b is in J. Suppose that $f : I \times J - \{(a,b)\} \to R$ is a function such that for all x in $I - \{a\}$ the limit exists $g(x) = \lim_{y \to b} f(x,y)$ and that for all y in $J - \{b\}$ the limit exists: $h(y) = \lim_{x \to a} f(x,y)$.

i. Show that even though the "iterated limits" may exist $L = \lim_{x \to a} g(x)$, $M = \lim_{y \to b} h(y)$, it may be the case that L does not equal M. Show, in thaqt case, the two-dimensional limit $\lim_{(x,y) \to (a,b)} f(x,y)$ fails to exist.

ii. Suppose in addition to the existence of the iterated limits one knows that the two dimensional limit exists: $f(x,y) \to N$ as $(x,y) \to (a,b)$. Show that then $L = M = N$.

SIXTH HOMEWORK ASSIGNMENT Due Friday, October 5, 2007.

8.1 [8, 10*, 11*], 8.2 [1, 2*, 4, 5, 6, 7*, 8, 10*, 11], 8.3 [1, 2*, 3, 5, 7, 8*, 11*]

SEVENTH HOMEWORK ASSIGNMENT Due Friday, October 19, 2007.

Read the review sections 8.3 and 8.4 about linear algebra. Do any problem whose solution isn't immediately clear. 8.4[15*], 8.5[10*], 9.1[1,3,5*,6*,7,8], 9.2[1*,3,4,5,8*,9*]

EIGHTH HOMEWORK ASSIGNMENT Due Friday, October 26, 2007.

9.3[1,3*,5,9*], 9.4[1,4,8*,11,12*,13*] (Problem 7 is not due until next week.)

A*. (See 9.3[8]) Suppose that (x,y,z) are the Cartesian coordinates of a point in R^3 and the spherical coordinates of the same point is given by

$x = r \cos \theta \sin \phi, \quad y = r \sin \theta \sin \phi, \quad z = r \cos \phi$

Let $u = f(x,y,z)$ be a twice continuously differentiable function on R^3. Find a formula for the partial derivatives of u with respect to x,y,z in terms of partial derivatives with respect to r, θ, ϕ. Find a formula for the Laplacian of u in terms of partial derivatives with respect to r, θ, ϕ, where the Laplacian is given by $\Delta u = \partial^2 u/ \partial x^2 + \partial^2 u/ \partial y^2 + \partial^2 u/ \partial z^2$.

NINTH HOMEWORK ASSIGNMENT Due Friday, November 2, 2007.

9.4[7*] (This problem is postponed from last week.) 9.5[2, 3*, 4, 6*, 9*, 12] (The problems from section 9.6 are postponed until next week.)

A*. Find the critical point (s_0,t_0) in the set $\{(s,t) \in R^2 : s > 0\}$ for the function with any real A and B,

$f(s,t) = \log(s) + [(t-A)^2 + B^2]/s$

Find the second order Taylor's expansion for f about the point (s_0,t_0). Prove that f has a local minimum at (s_0,t_0).

B*. Let $p > 1$. Find all extrema of the function $f(\boldsymbol{x}) = x_1^2 + x_2^2 + \ldots + x_n^2$ subject to the constraint $|x_1|^p + \ldots + |x_n|^p = 1$. If $1 \le p \le 2$ show for any $\boldsymbol{x}$ and n that

$n^{p-2}/(2p) \; (|x_1|^p + \ldots + |x_n|^p)^{1/p} \le (x_1^2 + x_2^2 + \ldots + x_n^2)^{1/2} \le (|x_1|^p + \ldots + |x_n|^p)^{1/p}$.

TENTH HOMEWORK ASSIGNMENT Due Friday, November 9, 2007.

9.6[1, 2*, 3, 7, 8*, 9]

A*. Let $F : R^2 \to R^2$ be given by $x = u^2 - v^2, \quad y = 2 u v$. Find an open set U in R^2 such that $(3,4) \in U$ and $V = F(U)$ is an open set, and find a C^1 function $G : V \to U$ such that $G \circ F (u,v) = (u,v)$ for all $(u,v) \in U$ and $F \circ G (x,y) = (x,y)$ for all $(x,y) \in V$. Find the differential $dG(F(3,4))$. (G is a local inverse. Solve for G and check its properties. Do not use the Inverse Function Theorem, which guarantees the existence of local inverse near $(3,4)$ assuming F is continuously differentiable near $(3,4)$ and $dF(3,4)$ is invertible.)

ELEVENTH HOMEWORK ASSIGNMENT Due Friday, November 16, 2007.

9.6[10, 11, 12*], 9.7[1*, 3, 5*, 7, 8*]

A*. In section 9.6 the Implicit Function Theorem was deduced from the Inverse Function Theorem. Show that the Inverse Function Theorem can be deduced from the Implicit Function Theorem.

TWELVTH HOMEWORK ASSIGNMENT Due Monday, November 26, 2007.

10.1[2, 3, 4, 5*, 6*, 8, 9*, 10], 10.2[1, 2*, 3, 4*, 5*, 6, 7, 8, 12*]

A*. Let $g : R \to R^2$ be a continuously differentiable curve. Show that the image of a compact interval $g([a,b])$ has Jordan content zero. (Called "volume zero" in the notes.) (Note that this would be false under the hypothesis of continuity only. This is because there exist "space filling curves." See, e.g., `http://www.math.ohio-state.edu/~fiedorow/math655/Peano.html`

THIRTEENTH HOMEWORK ASSIGNMENT Due Monday, December 3, 2007.

10.3[1, 2, 3, 4*, 5, 6, 7*, 9*, 10, 11*], 10.4[2*, 9*, 10*]

FOURTEENTH ASSIGNMENT Due Friday, December 7, 2007.

10.4[5, 6, 7*, 8, 12], 10.5[2*, 7*, 8*, 11*, 12*, 13]