

1. Find the eigenvalue λ of the matrix $A = \begin{bmatrix} -3 & 5 & 6 \\ 0 & -1 & 2 \\ -4 & 0 & 1 \end{bmatrix}$.

2. Let $\alpha_1, \alpha_2, \dots, \alpha_n \in \mathbb{R}$, where $n \geq 2$. Show that

$$\begin{vmatrix} 1 & \alpha_1 & \alpha_1^2 & \cdots & \alpha_1^{n-1} \\ 1 & \alpha_2 & \alpha_2^2 & \cdots & \alpha_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha_n & \alpha_n^2 & \cdots & \alpha_n^{n-1} \end{vmatrix} = \prod_{1 \leq i < j \leq n} (\alpha_j - \alpha_i).$$

Suggestion: Do it for $n = 2$ and 3 and then try to use induction on n .

3. Derive the following inequalities:

$$\begin{aligned} \sum_{j=1}^{\infty} |\xi_j \eta_j| &\leq \left(\sum_{k=1}^{\infty} |\xi_k|^p \right)^{1/p} \left(\sum_{m=1}^{\infty} |\eta_m|^q \right)^{1/q} \\ \left(\sum_{j=1}^{\infty} |\xi_j + \eta_j|^p \right)^{1/p} &\leq \left(\sum_{k=1}^{\infty} |\xi_k|^p \right)^{1/p} + \left(\sum_{m=1}^{\infty} |\eta_m|^p \right)^{1/p} \end{aligned}$$

4. Given a point $x_0 \in X$ and a real number $r > 0$, we define three types of sets:

$$B(x_0; r) = \{x \in X : d(x, x_0) < r\}$$

$$\tilde{B}(x_0; r) = \{x \in X : d(x, x_0) \leq r\}$$

$$S(x_0; r) = \{x \in X : d(x, x_0) = r\}$$

5.

Day	Min Temp	Max Temp	Summary
Monday	11°C	22°C	A clear day with lots of sunshine. However, the strong breeze will bring down the temperatures.
Tuesday	9°	19°	Cloudy with rain, across many northern regions. Clear spells across most of Scotland and Northern Ireland, but rain reaching the far northwest.

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